Duration Modelling of the After-Hours Electronic Futures Market

by

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Abstract

This thesis explores a class of models for modelling the time between trades, known as trade duration, in the after-hours electronic market for U.S. equity futures. These electronic markets have grown significantly over the last 10 years but little empirical work has been done to describe them. This is particularly so with duration modelling. High frequency trade duration data for the S&P 500 and NASDAQ-100 modelled in this thesis are collected from the GLOBEX electronic trading platform from the Chicago Mercantile Exchange for the period of 2004 to 2008.

This thesis first fits standard linear Autoregressive Conditional Duration (ACD) models with Exponential, Weibull and Generalized Gamma error distributional assumptions to the period 2004 to 2006. The Generalized Gamma distribution outperforms the alternatives but still provides unsatisfactory results in the form of serially correlated residuals (volume is used as an additional mark in the model specifications). In order to improve the models, nonlinear forms of ACD model are estimated. In particular, the threshold and logarithmic forms are implemented. Although the results improve with these more flexible forms, there remains continued evidence of nonlinearity in the results.

As a consequence, and taking into consideration the fact that the sample period of this thesis is much longer than the 3 month samples typically examined in the existing ACD literature, the thesis then examines the S&P 500 data for potential structural changes. Structural breaks are detected using a range of conditional Lagrange Multiplier tests associated with Andrews (1993) and Andrews and Ploberger (1994). Fitting Weibull ACD models to the segmented sub-periods identified with the structural break tests significantly improves the model estimation results.

Finally, this thesis examines the evidence for structural breaks in ACD models in the global financial crisis period using S&P 500 data from 2006 to 2008. The most significant structural change is found to occur in July, 2007, which is consistent with the onset of the crisis. Many of the structural change points detected in the data can be aligned with economic events during the crisis period, and sub-period estimations reveal the impact of the crisis on the electronic futures market.

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Overview and Motivation

Over the past decade or so, information technology-related developments have facilitated the acquisition and storage of all transaction (i.e. tick by tick) data and consequently, the availability of intraday and overnight financial databases. These ultra-high-frequency financial data have become invaluable to the study of the various issues related to the trading process and market microstructure, giving birth to the so-called high-frequency models. An inherent feature of such tick by tick data is that they are characterized by irregular spacing in time, as the transactions that generated these data may be clustered and/or scattered (Dacorogna et al. 2001). Short durations indicate heavy trading activity, high liquidity and new information. Longer durations indicate lack of trading activity, low liquidity and no new information. Newly developed market microstructure theory argues that durations convey valuable information and hence should also be modelled. Research work such as Goodhart and O'Hara (1997) and Madhaven (2000) suggest that the waiting time between trades plays an important role for understanding the processing of private and public information in financial markets. Diamond and Verrecchia (1987) and Easley and O'Hara (1992) also argue that traders may learn from the timing of trades by providing theoretical justifications for developing time series models of trade waiting times. Similar to other information such as price, volume, and bid-ask spread, duration should also be modelled.

Acknowledging the fact that the dynamic behaviour of durations contains useful information about intraday market activities, Engle and Russell (1998) proposed an autoregressive conditional duration model to describe the evolution of time durations. Zhang, Russell and Tsay (2001) extended the ACD model to account for non-linearity and structural breaks in the data. These duration models provide platforms on which duration related market microstructure theories are tested. Stylised facts of durations in high frequency financial data such as trade clustering and duration over dispersion can also be captured. In this thesis, a subset of linear and non-linear ACD models with and without structural breaks is applied to a large set of overnight data spanning the recent global financial crisis.

Meanwhile the trading environment in financial markets has changed dramatically over the last 10 years, with increasingly more instruments transacted electronically. Developments in electronic markets have lead to increased speed of trading and extended trading hours and locations. In particular, after-hours electronic markets have become increasingly popular in the past few years; both volume of trade and the number of market participants have increased dramatically. These newly developed after-hours electronic markets provide extended trading hours and unlimited trading locations, attracting investors worldwide.

Whilst many of the existing ACD models are based on short intervals of floor market data, there is very little, if any, duration modelling research based on the after-hours electronic markets. The question of how traders in after-hours electronic markets behave remains relatively unexplored. This motivates an application of duration modelling in this newly developed market in this thesis. Additionally, in contrast to much of the existing literature, which considers relatively short samples, the modelling is applied over a relatively longer period. This thesis provides empirical applications of duration models based on the after-hours electronic equity futures market for the S&P 500 and the NASDAQ-100 from July, 2004 to September, 2006 for the majority of the chapters. In the chapter 7, the S&P 500 data from October, 2006 to December, 2008[1] is examined during the global financial crisis.

As well as empirically fitting linear and logarithmic ACD models, this thesis contributes to the financial literature by studying structural changes based on the trade durations in the after-hours market during a non-crisis and crisis period. The thesis is essentially four potential papers presented as separate chapters following the literature review, market description, and overview of the global financial crisis chapters. Papers 1 and 2 presented as chapters 4 and 5 include linear and logarithmic applications of ACD models. Papers 3 and 4 study structural change effects under ACD modelling before and during the global financial crisis period, and are presented as chapters 6 and 7 respectively. Chapter 8 concludes.

[1] This 2006 to 2008 crisis period data sample is a continuation of 2004 to 2006 S&P 500 data sample.

1.2 Key Research Questions

1.2.1 ACD Modelling and the After-hours Electronic Market

The after-hours equity futures market studied in this thesis includes trading from all around the globe. The floor market and the after-hours market together trade almost 24 hours a day. Hence participants in these after-hours markets are able to adjust their positions almost immediately according to available information (news, events, and private information). As well as news announcement impacts from the extended trading hours, there are also geographical effects from remote traders worldwide. According to statistics from Global Exchange[2] (GLOBEX), 20% of the volume in the electronic market was transacted outside the US during 2009. The extended scope of trading drives a competitive and liquid futures market. Unlike the floor market, the fact that traders in the after-hours electronic markets are anonymous may also influence some investors' trading behaviour. This thesis fills a gap in empirically modelling after-hours electronic futures durations.

1.2.2 ACD Modelling and Error Distributions

This thesis applies linear ACD and log-ACD models based on Exponential, Weibull, and Generalized Gamma distributional assumptions on the error term. While much existing research work tends to build up an ACD model by allowing more general distribution assumptions on the error term, less research focuses on the

[2]GLOBEX is an electronic futures trading platform provided by the Chicago Mercantile Exchange.

actual distribution forms in the real data. These distribution forms can differ across different type of markets. In this thesis, ACD models with each of the above three error distribution assumptions are estimated and compared.

1.2.3 ACD Modelling Under the Global Financial Crisis

Most existing ACD model literatures are based on relatively short intervals of duration data in normally behaved market conditions. Many theoretical developments in ACD models, such as IBM data in Engle and Russell (1998) and three NYSE securities data in Bauwens and Giot (2000), are also based on data samples within normal market conditions. Research has also been carried out to explore how to extend ACD models by relaxing distributional restrictions rather than on extended samples. For example, the Stochastic Conditional Duration (SCD) model in Bauwens and Veredas (1999) and the Stochastic Volatility Duration (SVD) model in Ghysels, Gourieroux, and Jasiak (2004). However, in the real world market conditions may be changing due to news, events, and market uncertainties, the empirical implementations of the more flexible but complicated ACD models may be difficult. How ACD models perform under volatile market conditions, such as the recent 2007-2008 global financial crisis, still remains unclear. The issue of how the clustering behaviour of durations is captured in this type of market during crisis events is addressed in chapters 6 and 7 of this thesis.

1.3 Thesis Structure

This thesis is presented in eight chapters. Following the introduction in chapter 1, chapter 2 provides a review of duration models and structural break studies.

The review begins with issues concerning why researchers study durations. The discussion of various forms of ACD models proceeds from linear to nonlinear models. Linear forms of ACD models are closely related to the original duration model of Engle and Russell (1998). Many extensions have followed from the strong similarity between ACD models and GARCH models, such as Generalized Gamma ACD from Lunde (2000), logarithmic ACD from Bauwens and Giot (2000), and threshold ACD from Zhang, Russell, and Tsay (2001).

There are three major types of nonlinear ACD models discussed in the literature review, namely the regime-switching, logarithmic, and latent factor-based classes. The developments in distributional form assumptions of the error term are then presented. Following the introduction of different forms of ACD models, tests to examine their performance are discussed. These tests are presented in the order in which problems are encountered with the estimation misspecifications. Some existing applications of ACD models based on trade durations, price durations, and volume durations are then presented.

After the review of ACD models, a brief review of the developments of structural break tests in conditional models is then presented. The focus is on break tests based on multiple unknown locations, such as the Andrews and Ploberger (AP) Lagrange Multiplier based tests and CUSUM type tests from Inclan and Tiao (1992). The AP LM based tests methodology for detecting breaks are primarily introduced since these tests are applied in chapters 6 and 7 of this thesis.

Following the literature review in chapter 2, a description of the after-hours electronic futures market and an overview of the 2007-2008 global financial crisis are given in chapter 3. The first part of chapter 3 documents developments in electronic trading through the GLOBEX platform, including information about how the market operates, and the products available on this after-hours trading system. The second part of proceeds an overview of the 2007-2008 global financial crisis. It outlines the background, build-up, and global consequences of the crisis. More details related to the crisis events are studied in chapter 7.

Chapter 4 marks the beginning of the empirical applications of ACD models in this thesis. In this chapter, linear ACD models with Exponential, Weibull, and Generalized Gamma distributional assumptions on error terms are applied. These models are pursued to the highest lag orders which produce convergence. The long memory pattern in the data is found through its autocorrelation function and lags length within the model. This chapter shows that applying more general forms of distributional assumptions and allowing higher lag orders, results in ACD models which fit the sample data better.

The role of volume within the ACD models is then explored in chapter 4. Using volume as an additional mark in the model exposes a significant negative relationship between volume and expected duration. At the end of this chapter, a threshold ACD model is applied to the S&P 500 data as a first attempt at a nonlinear ACD model. Compared with results from the linear ACD models, the threshold model obtains much improved estimates. This implies the necessity of exploring nonlinear modelling for the after-hours market data set.

In chapter 5, as an attempt at a nonlinear ACD modelling, the logarithmic ACD models are applied to the S&P 500 data. Compared with other nonlinear forms of ACD models described in the model reviews of chapter 2, the logarithmic ACD model is far less costly to estimate. Two types of log-ACD modes are presented in the chapter based on the assumptions on the error function within the model. Over-dispersion ratios are presented before and after the estimates.

An improved Weibull log-ACD (2,2) model with volume as additional mark is obtained in this chapter.

Considering the data sample studied in this thesis is very long (more than 2 years), in subsequent chapters we study possible structural changes. In chapter 6, the Andrew and Ploberger Lagrange Multiplier based tests are applied to the sample data set in order to locate multiple unknown break points within the sample. A Weibull ACD(1, 1) model is first fitted to the sample data, and the obtained model derivatives are then used to perform the break tests, which divides the data sample into a number of sub-periods. Each of the sub-periods is then estimated using a simple Weibull ACD (1,1) model. The ACD parameters and summary statistics for each individual sub-periods are then presented; and a comparison of the sub-periods shows that different duration patterns are evident. This build-up leads to a more detailed structural changes study over the financial crisis period in chapter 7.

Following the structural breaks detection framework of chapter 6, the S&P 500 data traded on the GLOBEX after-hours market over the financial crisis period is studied in chapter 7. The investigation of duration during the 2007-2008 global financial crisis provides key information on the progress of the crisis. This sample is complicated for the follow two reasons. First, in early 2007 CBOT and CME merged, and secondly later in the same year the global financial crisis began, making it highly possible that this volatile sample data experiences multiple structural breaks. The most significant change point identified over the October 2006-December 2008 period is found on 24th, July 2007, a point which aligns perfectly with anecdotal assessments of the crisis as beginning in 'mid July 2007'. Many of the major breaks points are aligned with economic events during the same period.

Weibull ACD (1,1) models are then applied to each of the sub-periods detected to study the individual diurnal patterns. Distinctive duration behaviours are found across different sub-periods. The relationship between economic events and the ACD models are then tabulated, by studying ACD model parameter changes across event dates. The changing forms of distribution for the data as the crisis develops are also discussed in this chapter. The structural breaks detected from ACD models in this thesis strongly indicate that durations are valuable information which help to identify the market microstructure.

Finally chapter 8 concludes with the findings and future suggestions originating from this thesis. Results summarized from the linear and nonlinear ACD models indicate strong nonlinear patterns in the after-hours market. The after-hours electronic market data set examined in this thesis experienced a large number of structural breaks, and by allowing explicitly for structural breaks in the longer and more volatile data set does improve overall estimates. The individual market dynamics are better captured and explained by modelling duration under different market conditions.

Chapter 2

Literature Review

2.1 Introduction

Market microstructure theory has received increased attention since the rapid developments in high frequency data analysis in financial markets. As suggested by papers such as Easley and O'Hara (1992) and Diamond and Verrecchia (1987), the time between trades may reveal information and should be modelled. The ACD model framework used in this thesis brings forward the necessity of a review of literature on duration modelling.

Section 2.2 discusses developments in high frequency data and ACD models. In addition to the introduction of the standard ACD model from Engle and Russell (1998), extensions of ACD models, ACD model testing, and ACD model applications are also presented in section 2.2. Motivated by the structural effect studies in chapter 6 and 7 of this thesis, a review of structural break studies under ACD framework is presented in section 2.3.

2.2 Market Microstructure Theory and Duration

Recent market microstructure theory, such as in Easley and O'Hara (1992), argues that the time between trades may reveal information and should be modelled. Diamond and Verrecchia (1987) also provide theoretical justifications for studying time series models of durations. They argue that durations are important in leading markets to price discovery with different levels of information about the value of traded assets. The same paper suggests that longer durations lead to negative adjustment of asset values, suggesting that long durations are more likely when informed traders are selling an asset, but with a short-sell constraint limiting them from doing so. On the other hand, Easley and O'Hara (1992) suggest the long durations imply that the value of the particular asset has not changed, corresponding with uninformed market traders, and short durations imply the existence of asymmetric information with a high level of trading activities. The same paper also suggests that spreads will decrease when the time between transactions increases. As the absence of trades implies that no news arrives, and therefore the likelihood of informed trading decreases, the adverse selection component of the spread declines.

According to Easley and O'Hara (1987), a high trading rate could include trades broken up from large volume trades by informed traders. Market microstructure papers such as Glosten and Milgrom (1985), Hasbrouck (1988), Goodhart and O'Hara (1997), O'Hara (1995), and Madhaven (2000) suggest that the durations of trades, order arrivals and price changes play important roles in understanding the processing of private and public information in the market. Easley et al. (1996) claim information such as the probability of informed trading can be extracted from duration modelling. Other market microstructure papers such as Diamond and Verrecchia (1987), Easley, Kiefer and O'Hara (1997), and Hasbrouck (1991) also suggest waiting time between consecutive trades provides valuable information and should be modelled. In the context of irregular spaced duration time series, traditional time series models are technically no longer applicable, and aggregating the data into regular intervals may lead to loss of information.

Intraday high frequency data often exhibits intraday seasonality. For example, Engle and Russell (1998) show the inverted-U shaped daily pattern for durations, which represents a higher level of trading activity at the start and end of a trading day in the floor market. Dungey, Fakhrutdinova, and Goodhart (2008) also found a complex diurnal pattern in the after-hour electronic market. These patterns provide information on the features of the exchange and traders' behaviours. For example a high level of overseas trading activities can be recognized from their time zone in this diurnal pattern, since the after-hour electronic market offers trades all over the world. The overnight information plays an important role as traders prefer to benefit as quickly as possible from their information. It often appears as high volume of trading before the floor markets open in the morning, as investors take advantages of their private information. The raw duration is assumed to be a multiplicative function of a stochastic series (the adjusted duration) and a deterministic pattern (daily seasonality). In high frequency data modelling, the daily pattern is often removed to ensure an unbiased estimation. A popular method for removing the diurnality is using a spline following Engle and Russell (1998). In later chapters, all the durations are diurnally adjusted.

2.3 Types of Duration Models

2.3.1 Linear Form ACD Models

A distinctive feature of intraday high frequency data is that the time intervals between each transaction are irregular. These irregularly spaced durations are often treated as a point process, which is defined as a stochastic process that generates a random list of points on the time axis (Bauwens and Giot, 2001). If the duration x is a realization of the random variable X, the distribution function of X can be written as $F(X) = \Pr(X \leq x)$. A survival function captures the probability that some specific event will survive beyond a specific time. A survival function is defined mathematically as $S(X) = \Pr(X \geq x)$ or simply $1 - F(X)$, since in general $F(X)+S(X) = 1$. The density function is defined as $f(x) = \frac{dF(x)}{dx}$, or it can be rearranged to the form $f(x) = \frac{-dS(x)}{dx}$. A hazard function is the ratio of the density function and the survival function, $h(x) = \frac{f(x)}{S(x)}$. In high frequency financial data, a hazard function indicates the probability that a duration ends given that it lasts as long as x. In order to capture the distribution of the error term in a ACD formation, ε_i, the concept of a baseline hazard is introduced. A baseline hazard is the hazard function of ε_i, defined by the density function of ε over the associated survival function:

$$\lambda_0(t) = \frac{p_0(t)}{S_0(t)} \tag{2.1}$$

The conditional intensity can be then written as:

$$\lambda(t \mid N(t), t_1, ...t_{N(t)}) = \lambda_0(\frac{t - t_{N(t)}}{\psi_{N(t)+1}})\frac{1}{\psi_{N(t)+1}} \tag{2.2}$$

Although durations have been studied since the 1950s via survival analysis, the first to model the clustering feature of times between trades is the ACD model introduced by Engle and Russell (1998). The ACD model provides a generalized autoregressive framework for conditional durations. The intraday duration data are treated as a collection of events and the time of the event represents the time it occurred in a point process. Apart from the time between trades, other trading information such as price, volume and spread can also be added as additional marks in an ACD model, and the point process then becomes a marked point process.

Since past dynamics affect the rate of arrivals, the ACD model falls into the class of accelerated failure time model (AFT). The expectation of the *ith* duration x_i is given by:

$$\psi_i \equiv E(x_i \mid x_{i-1}, ..., x_1) = \psi_i(x_{i-1}, ..., x_1; \theta), \tag{2.3}$$

where θ is the parameter set. In the general form of an ACD(p,q) model, the duration x_i is assumed to be the product of the expected duration and an independent and identically distributed (*i.i.d.*) error term. The conditional expected duration is assumed to follow an autoregressive conditional process such as:

$$\begin{cases} X_i = \psi_i \varepsilon_i \\ \psi_i = \omega_0 + \sum_{j=0}^{p} \gamma_j x_{i-j} + \sum_{j=0}^{q} \omega_j \psi_{i-j}, \end{cases} \quad \varepsilon_i \sim i.i.d. \tag{2.4}$$

where ω_0 is a constant, γ_j and ω_j model parameters with $\sum \gamma_j + \sum \omega_j < 1$. ACD models are often treated as counterparts of GARCH models. The duration clustering captured in ACD models is similar to the volatility clustering in GARCH models.

ACD models may incorporate many alternative assumptions on the distributional form of the error term ε_i. The simplest form of ACD model is an exponential ACD (EACD) (1,1) model from Engle and Russell (1998). It shares many similarities with the GARCH model introduced by Bollerslev (1986). Similar to clustered volatilities, clustered trade arrivals have also been found by much recent evidence and these clusters of trading may occur from information-based clustering or liquidity-based clustering. As in Engle and Russell (1998), Equation (2.4) can be rewritten into an ARMA form by introducing a martingale difference, which further suggesting strong similarity between ACD and GARCH models. Engle and Russell (1998) also considered the assumption that the ACD model error term follows a standard Weibull distribution where the scale parameter is set to be one. This model is called Weibull ACD (WACD) model. In this case the conditional hazard function is solely determined by the shape parameter. If the shape parameter is larger (smaller) than one, there is an increasing (decreasing) conditional hazard function. For exponential ACD, the conditional hazard function is flat since the shape parameter is one. The choice of the error term distribution makes a huge difference in ACD models and has great importance, since the expected duration enters the conditional heteroskedasticity equation as explanatory variables.

Drost and Werker (2004) found consistent estimates when QML estimation is based on the standard Gamma family distribution. Since the Exponential and Weibull distribution both belong to the Gamma family, quasi-maximum likelihood (QML) estimators can be obtained for the ACD parameters by using both Exponential and Weibull distributions. The same property also applies to the

Generalized Gamma distribution ACD (GGACD) model from Lunde (2000). In practice, QML might be easier to implement but less efficient, whereas a fully efficient maximum likelihood (ML) estimate is often more efficient and preferred. Grammig and Maurer (2000) suggest QML estimates perform poorly in finite samples and that QML with monotonic hazard rates yield unsatisfactory results even with very large sample sets. When the true data generating process involves non-monotonic hazard functions, the QML estimators of the standard ACD Model are less efficient and biased. When the estimators are biased, the predicted expected durations also tend to be biased.

Grammig and Maurer (2000) argue that the assumption of monotonic hazard functions maintained in the standard ACD specifications is too restrictive. They introduced the Burr-ACD model which assume a Burr distribution for the error term. A Burr distribution can be treated as a Gamma mixture of Weibull distributions; its special cases lead to Exponential, Weibull and Log-logistic distributions. The hazard functions for the Generalized Gamma and Burr distribution are both hump-shaped as determined by their two shape parameters. As suggested by Grammig and Paurer (2000), ACD models allowing for non-monotonic hazard functions are clearly in favour compared to standard ACD models with monotonic hazards.

Apart from the above, there have also been many other distributional form assumptions of the error term, for example Hautsch (2002) uses a Generalized F distribution, De Luca et al. (2004) use a mixture of two Exponential distributions based on information and agents. Both papers find their model fits intraday data very well. Bauwens, Giot, Grammig, and Veredas (2000) suggest that the standard ACD models fail to model observations in the tails of their distribution and often yield unsatisfactory performance in forecasting. All the above arguments against the standard ACD model pushed the development of ACD models forward.

Extended ACD models are based on the analogy between ACD and GARCH models. Following the idea of the Fractionally Integrated GARCH (FIGARCH) model from Baillie, Bollerslev, and Mikkelsen (1996), a Fractionally Integrated ACD (FIACD) model was proposed by Jasiak (1998) to capture the well-known long memory phenomenon in the financial high frequency data. More recently, Dungey, Henry, and McKenzie (2008) also investigate US Treasuries using a FI-ACD model. However, since the long-memory of the high frequency data could also be caused by structural breaks and different regime effects, there is an outstanding issue about the efficiency of the FIACD model. Fernandes and Grammig (2006) introduce the augmented ACD (AACD) model, following the approach taken by Hentschel (1995), who develop a class of asymmetric GARCH models.

Empirical studies such as Dufour and Engle (2000a), Zhang, Russell, and Tsay (2001), and Fernandes and Grammig (2001) suggest that the linear specification of ACD model is not sufficiently flexible to capture the adjustments in the duration process. Nonlinearity is an important characteristic of duration data, particularly when we want to link real economic events with the durations. The following section describes the properties of some of the nonlinear ACD models.

2.3.2 Regime-switching Class of ACD Models

As nonlinearity has been a common issue in financial time series data, it is not surprising that there is evidence of nonlinearity in duration processes addressed in papers such as Dufour and Engle (2000a) and Zhang, Russell, and Tsay (2001).

Engle and Russell (1998) found that even a 3-month period of IBM intraday duration data experiences significant nonlinearity. Later research in Ghysels, Gourieroux, and Jasiak (2004) found nonlinear ACD models may be more appropriate than linear specifications for modelling high frequency financial data. One approach has been to divide the duration process into different regimes according to different thresholds or filters. A k-regime TACD(1,1) model follows:

$$\begin{cases} x_i = \psi_i \varepsilon_i^k \\ \psi_i = \omega_0 + \gamma_1^k x_{i-1} + \omega_1^k \psi_{i-1}, \end{cases} \quad \text{if } x_i \in R_j \tag{2.5}$$

where R_j is a matrix of threshold values for each regime. As in the general form of ACD model in equation (6.11), the $\{\varepsilon_i^{(k)}\}$ are also assumed to be drawn from an *i.i.d.* process with positive density function $f^{(k)}(\cdot)$ with $\{\varepsilon_i^{(k)}\}$ set to be 1. The error terms ε_i in each sub-regime are independent.

Zhang, Russell, and Tsay (2001) use a three-regime threshold ACD (TACD) model to capture the nonlinear relation between the conditional expected duration and lagged durations. The threshold values are obtained by treating the duration process as a self-exciting threshold autoregressive process, and using a grid search algorithm to locate the optimum combination of the grid thresholds that maximize the conditional likelihood. Although the grid search algorithm is computationally demanding when the data set is large, Zhang, Russell, and Tsay manage to locate 3 significant regimes and identify a fast trading regime as the informed regime and a slow trading regime as uninformed trading regime.

Zhang, Russell, and Tsay (2001) identify 6 structural breaks in a 3-month sample of IBM data and divide their data into 7 sub-periods. The break points located can be closely aligned with real economic events and each sub-period divided is fitted with a separate TACD model. Their model shows different characteristics in each different regime. By looking into the nonlinear dynamics, they argue that the TACD model gives a better understanding of the correlated waiting times between transactions. More literature regarding to detection of structural breaks is shown in the next section of this literature review (Section 3).

While the threshold ACD models are able to provide improved estimates over sub-period duration processes, their transition from one regime to another involves a jump process and is not smooth. Literature on smooth transition autoregressive (STAR) processes covers the transition problem. The smooth transition ACD model can be treated as an alternative to the threshold ACD model and they are very closely related. A transition function is assumed to be a logistic function to make the transitions between different states. Granger, Teräsvirta, and Anderson (1993), Teräsvirta (1994) and Teräsvirta (1998) study the smooth transition for the conditional mean and Gonzales-Rivera (1998) and Lundberg and Teräsvirta (2002) study for the conditional variance. Following a similar idea, Meitz and Teräsvirta (2006) introduced the smooth transition ACD (STACD) and the time-varying ACD (TVACD) models. The TVACD model considers the situation where the data sets last relatively long periods. In this case certain events and new economic environments might change the structure of the time series process. Thus, having a fixed set of parameters over such a period may no longer be appropriate. Solutions to this problem could lead to dividing the data sample into sub periods and estimating individually as in Zhang, Russell, and Tsay (2001), however this involves heavy computational work to locate the number and timing of the break points. The TVACD model allows the parameters to change smoothly over time, with a logistic transition function and time is its transition variable.

Another type of regime-switching ACD model is the Markov switching ACD (MSACD) model proposed by Hujer, Vuletic, and Kokot (2003). The MSACD models make use of an unobservable stochastic process which follows a Markov chain. The idea of MSACD model in Hujer, Vuletic, and Kokot (2003) is based on the Expectation-Maximization (EM) algorithm in Dempster, Laird, and Rubin (1977). They advocate that a Markov switching model can be based on non-Gaussian marginal distributions. MSACD model is suggested to be a better description of duration process than the standard ACD models in Engle and Russell (1998). Hujer, Vuletic, and Kokot (2003) show that the MSACD is also a better forecasting tool for time series of durations, and it yields better forecast performance than linear ACD models. The MSACD model results for trade durations are shown to be consistent with the market microstructure model in Easley, Kiefer, O'Hara, and Paperman (1996). The literature in the regime-switching ACD models confirms that nonlinear ACD specifications are more powerful and appropriate to model the financial duration process than linear specifications.

2.3.3 Nonlinear Logarithmic ACD Models

One popular nonlinear extension of the standard ACD model is a more flexible logarithmic-ACD (log-ACD) model as proposed by Bauwens and Giot (2000). One of the major advantages of the log-ACD model is that it avoids the negative durations problem caused by possible negative coefficients. Hence the log-ACD model requires far fewer restrictions on the model specification. The log-ACD specification is also able to model the next conditional mean duration asymmetrically with shorter and longer durations of conditional mean respectively. Two forms of log-ACD models (log-ACD$_1$ and log-ACD$_2$) exist, differentiated by the form of the error term function. For log-ACD$_1$ model, the error function assumes $g(\varepsilon_{i-j}) = \ln x_{i-j}$, where the log-ACD$_2$ model assumes the error function follows $g(\varepsilon_{i-j}) = \varepsilon_{i-j} = \frac{x_{i-j}}{e^{\Psi_{i-j}}}$. In fact, the log-ACD$_2$ specification seems to fit high frequency data better overall and is often preferred.

Maximum likelihood is used to estimate log-ACD models. The unconditional moments and the autocorrelation functions of the log-ACD model are studied in Bauwens, Galli, and Giot (2003). According to Bauwens, Giot, Grammig and Veredas (2000), the log-ACD model implies a non-linear relation between the duration and its lags. The same paper compares different ACD models via density forecasts, and finds that the threshold models do not necessarily outperform the log-ACD models. Log-ACD models can be also further classified according to the error distribution forms. For example, Bauwens et al. (2008) study the Exponential, Weibull, Gamma, Burr and Generalized Gamma forms of log-ACD models.

2.3.4 Latent Factor-based Duration Models

Some literature treats the conditional duration as a latent variable instead of a deterministic variable. In the GARCH literature, it is well known that compared with GARCH frameworks, using unobserved latent variables in stochastic volatility (SV) modelling yields favourable results, and the dynamics of the financial time series are better captured. The advantages of SV models over GARCH models were discussed in Danielson (1994), Kim, Shephard, and Chib, (1998), and Ghysels, Harvey, and Renault (1996).

Based on the similarities to GARCH models, Bauwens and Veredas (1999) introduced the Stochastic Conditional Duration (SCD) model. The SCD model follows:

$$\begin{cases} x_i = \psi_i \varepsilon_i \\ \ln \psi_i = \omega + \beta \ln \psi_{i-1} + u_i, \end{cases} \quad |\beta| < 1 \tag{2.6}$$

where the two error terms $\varepsilon_i \sim i.i.d$ and $u_i \sim i.i.d$ in equation (2.6) are the two sources of unobservables in the observed and conditional durations. The unobserved latent variables in the SCD model can be treated as information flows driving the duration process that cannot be observed directly. The second line of Equation (2.6) can be treated as a stationary autoregressive process. The parameter of the lagged log conditional duration, β, is forced to be less than unity. Compared with traditional ACD models, the latent variables in SCD models can yield more complex shapes for the hazard functions. Also the distributional assumptions for ε_i and u_i can be different, which makes the SCD model a mixture model. The above conditions give greater flexibility in modelling the dynamics of the duration process and also make it possible for the SCD model to capture the unobservable information in the market. Bauwens and Veredas (1999) also find the SCD model yields favourable results in a comparison study with the log-ACD model. However, the more complex assumptions make the exact likelihood function very difficult to locate and time consuming to estimate. The multidimensional integral requires heavy simulations which are especially extensive when the data set is large. Bauwens and Veredas (2004) use QML methods with the Kalman filter[1], but again this has the problem of estimate efficiency since it is not using the true likelihood of durations.

[1] An alternative is the Monte Carlo Markov Chain (MCMC) technique in Strickland, Forbes, and Martin (2006)

Ghysels, Gourieroux, and Jasiak (2004) propose a more flexible Stochastic Volatility Duration (SVD) model, which allows one to estimate the dynamics for the conditional variance. Note that the traditional ACD model does not allow for independent dynamic parameterization for the conditional mean and conditional variance. The reason for this restriction is that the traditional ACD model assumes that the higher order conditional moments are directly linked to the first moment of conditional mean. Ghysels, Gourieroux, and Jasiak (2004) indicate that this assumption is too restrictive, and information from variance such as market liquidity and risk could be lost. The SVD model however, combines the dynamics of the conditional mean and conditional variance with two time varying factors in the model. The SVD model starts from two independent Gaussian random factors, and analyses the conditional mean and conditional variance dynamic patterns using a VAR representation. The initial SVD model is build on standard exponential duration model with gamma heterogeneity from cross-sectional and panel data literature. It assumes duration $x_i = \frac{U}{aV}$, where U follows an exponential distribution with intensity one, V follows a gamma distribution and is independent of U. In terms of Gaussian factors, the SVD model can be expressed as:

$$x_i = \frac{H(1, F_1)}{aH(b, F_2)}, \tag{2.7}$$

where a and b are positive parameters, $F1$ and $F2$ are i.i.d. standard normal variables, and $H(b, F) = G(b, \Phi(F))$ where Φ is the c.d.f. of the standard normal distribution and $G(b, \cdot)$ is a quantile function of Gamma (b, b) distribution.

Since the SVD model belongs to the family of nonlinear ACD models, its likelihood function is difficult to evaluate. In fact there have been few applied studies of SVD models in the literature. Ghysels, Gourieroux, and Jasiak (2004) use a two-step procedure, which first assumes the marginal distribution of duration x_i follows a Pareto distribution depends only on a and b, and then uses simulated moments method to obtain the autoregressive parameters. Strong dynamics in both conditional mean and variance factors are found using Paris Stock Exchange data in Ghysels et al. (2004). Note that Bauwens, Giot, Grammig, and Veredas (2004) argue that the assumption of Pareto distribution in the first step of the above procedure might not be appropriate. The forecasting performance of the SVD is also found to be very poor compared with traditional ACD and log-ACD models in Bauwens et al. (2004).

Another latent variable based models is the discrete mixture ACD (MACD) model of Hujer and Vuletic (2004). Instead of treating the duration process as linear and following a particular form of distribution, they combine the idea of mixture models and ACD models. In common with other latent variable ACD models, the introduction of a discrete-valued latent regime variable increases the flexibility of the model, in order to capture the specific characteristics of intraday duration data. The discrete mixture ACD model proposed by Hujer and Vuletic (2004) can also be viewed as a compromise of the two extreme models of Markov switching ACD in Hujer, Vuletic,and Kokot (2003) and the discrete mixture exponential ACD model in De Luca and Gallo (2004).

Overall, the ACD literature is moving forwards in favour of the nonlinear models. The more comprehensive models give greater flexibility and better understanding of the market information. However, the problem of evaluating the more complicated likelihood function is yet to be solved. Further research on ACD models is still needed.

2.4 ACD model Testing

Not only the optimal specification of ACD models is far from finalized, but also there are issues about how to examine their adequacy. To date most of the papers in the ACD literature only use simple examinations of the standardized residuals. However, within the limited papers on this issue, there are some helpful procedures proposed for examination of an ACD model. A few approaches are briefly reviewed in the following paragraphs.

The most obvious approach to examine the goodness of fit of an ACD model is to check the dynamical and distributional properties of the standard residuals from the model. If the ACD model is correctly specified, the residual series should be *i.i.d.* The standardized residuals follow:

$$\hat{\varepsilon}_i = x_i/\hat{\psi}, \quad i = 1, ..., n.$$

Checking the Ljung-Box Q-statistics is a common approach which has been adopted by many papers following Engle and Russell (1998). Some use Box-Pierce statistics to check whether the temporal dependence has been captured by the underlying ACD model. Graphically, Quantile Quantile-plots are also used to check the standardized residuals, as presented in Bauwens and Veredas (2004) and De Luca and Gallo (2004). Other papers including Li and Mak (1994) and Bauwens and Giot (2000) suggest this approach of checking the serial correlations in the residuals is very questionable.

Other approaches have also been proposed apart from examining the residuals. Under an ACD model, many things can go wrong and cause misspecifications. Some common sources are: an incorrect functional form of the conditional mean; an inappropriate distributional form for the error term; failure to account for nonlinearity; possible higher-order ACD models; inappropriate use of QML estimation, and inconstancy of the model parameters.

One approach to test the functional form is to fit an ACD specification into a more general form and use the Lagrange Multiplier (LM) test. The LM test is useful to test model misspecifications. Meitz and Teräsvirta (2006) develop a powerful LM type test which can be used to test against higher order models, linearity, and parameter constancy types of misspecification in the conditional mean functional form.

Other tests have been developed to test the misspecification of an inappropriate distributional form of the error term. For example Fernandes and Grammig (2005) propose the D-test to consider the distance between the parametric density function and nonparametric estimate, and the H-test to map the hazard rate. Diebold, Gunther, and Tay (1998) introduce a test framework which evaluates the density forecasts of the ACD model. If the density forecast is correct, the probability integral[2] transforms of the density forecast should be *i.i.d* uniform under the null hypothesis. A rejection of the null hypothesis indicates a misspecification of the goodness of fit but the test does not show the cause of the rejection. This test is also used in Bauwens, Giot, Grammig, and Veredas (2004). Engle and Russell (1998) divide their duration data into a number of bins from $0 \sim \infty$ and regress the estimated residuals against its previous duration. If the estimated residuals are truly *i.i.d.*, the regression coefficient of determination should be zero. This approach is designed to detect nonlinear dependencies of

[2]The probability integral transform follows: $q_i = \int_{-\infty}^{x_i} f(s)ds$. see Gunther et al (1998) for more details.

ACD models, and is also adopted by Zhang, Russell, and Tsay (2001).

Overall, more tests to examine the adequacy of the ACD models are yet to be developed. Considering various sources of model misspecification, to fully test a model requires several of the above individual tests, which each involves a significant amount of empirical work. The LM test in Meitz and Teräsvirta (2006) examines some misspecifications but a more general form of test is still missing to fully evaluate ACD models efficiently.

2.5 ACD Model Applications

Over the last decades, ACD models have been primarily used to evaluate trade durations and price durations. Few have considered volume durations and other economic events. In the following subsections, review of these different types of ACD model applications are presented.

2.5.1 Trade Duration Applications

Trade duration is simply defined as the time difference between two consecutive trades. It is the type of duration that has been mostly applied in the literature. The main features of trade duration found in most papers are that trade duration is often clustered and experiences over-dispersion. The clustering phenomenon of the trade duration, where long (short) durations are likely to be followed by long (short) durations, is often related to information arrivals in the market. The autocorrelation functions (ACF) of the trade duration in most papers suggest slow decreasing persistence. Some papers such as Jasiak (1998), Engle and Russell (1998), and Bauwens et al. (2004) show long memories behaviours for the trade durations. The sum of ACD coefficients is very close to unity. Another feature of the trade duration is that it is often found to be over-dispersed (with its standard deviation is larger than its mean). A dispersion ratio is used in Bauwens et al. (2008) and finds evidence of over-dispersion in 5 selected stocks data from New York Stock Exchange. A dispersion ratio is defined as standard deviation/mean of a time series. An over-dispersion means a distribution with a higher than expected variance. It basically implies that there is more variability around the model's fitted values. Dispersion problem is more often cited in the literature for generalized linear models[3]. The over dispersion test is also used in ACD models as one of model diagnostics, for example in Engle and Russell (1998). Many of the findings in trade duration applications such as Engle and Russell (1998), Zhang, et al. (2001), and Fernandes and Grammig (2006) suggest difficulties in fully removing serial correlations in the residuals.

A major issue dealing with trade duration is how to treat zero durations. As markets become more liquid, many transactions take place at the same time (down to per second accuracy). Most papers follow the approach in Engle and Russell (1998), which simply aggregates the simultaneous transactions into one. All other transactions are deleted and volume occurred at the same time are aggregated. Many researchers argue that by deleting the intervening transactions information can be lost. Zhang, et al. (2001) investigate the effects of multiple simultaneous transactions and show that it is not the number of transactions at the time that matters, it is the occurrence of zero durations. However, Veredas, Rodriguez-Poo, and Espasa (2001) argue that the occurrence of zero durations may be caused by traders posting limit orders to be executed at the same price.

[3]For more information, see Dean (1998) and Cox (1983).

The same authors argue that by deleting the simultaneous trades the dynamics of the trade duration may be changed. Bauwens (2006) find higher Ljung-Box statistics and residual correlation when zero durations are removed. This is an interesting issue and not many studies have been conducted on this matter. It is especially more complicated for the after-hours electronic markets, where trades follow price-time priority rules. In such cases zero durations could be raised from many factors, including input/process delay and the electronic platform's matching logic.

2.5.2 Price Duration Applications

Price duration is defined as the time taken for a fixed change in an asset's price. Engle and Russell (1998) show that the conditional hazard function of price duration is closely linked to the instantaneous volatility of price. The diurnal pattern for price duration can be treated as intraday volatility diurnal pattern following the inverse relationship between price duration and volatility. Papers such as Bauwens et al. (2004) and Fernandes and Grammig (2006) find it is relatively easy to fit an ACD model on price durations, and to remove serial correlations in the model residuals. Bauwens et al. (2004) also found that for price durations, more complicated nonlinear ACD models such as Threshold-ACD and stochastic volatility duration models do not necessarily outperform the standard ACD and log-ACD models.

Most of the market microstructure theory on price durations relate to volatility and risk. Compared with trade durations, price durations are far less studied. Some applications include Engle and Lange (2001), Prigent, Renault, and Scaillet (2001), Gerhard and Hautsch (2002), and De Luca and Gallo (2004).

2.5.3 Volume Duration Applications

Volume duration on the other hand is defined as the time consumed to achieve a given level of aggregated volume for an asset on the market. It was introduced by Gourieroux, Jasiak, and Le Fol (1999). Since volume duration is a measure of volume changes it is also helpful to measure liquidity. Only a handful of ACD applications have been conducted on volume durations, including Bauwens et al. (2004), Bauwens and Veredas (2004) and Fernandes and Grammig (2006). Bauwens et al. (2004) suggest log-ACD models with a flexible error distributional form (such as Burr and Generalized Gamma) perform very well on price and volume durations, but the EACD and SVD perform badly.

2.5.4 Other Types of Duration Applications

Apart from stock market transaction durations, duration modelling have been applied in wide areas in economic activities. For example, Fischer and Zurlinden (2004) study the duration between central banks interventions on foreign exchange market; Focardi and Fabozzi (2005) apply duration modelling in credit risk analysis by treating defaults in credit portfolio as a point process; Rossi, Noè, and Sianesi (2008) use ACD models to capture the continuous process dynamics on discrete manufacturing sub-systems in the fibre-glass industry.

2.6 ACD Models and Structural Breaks

The purpose of presenting an overview of the structural break literature is that two of the chapters in this thesis involve modelling ACD models with structural breaks. The above ACD model literature also indicates that the nonlinear ACD models are mostly preferred over the linear ACD models. Some popular tests for nonlinearity are the F-test by Tsay (1986), augmented F-test from Luukkonen, Saikkonen and Teräsvirta (1988), threshold test by Tsay (1989) and general nonlinearity test by Tsay (1991). There have been tremendous developments in structural break research since the 1990s, especially in detecting unknown location structural breaks with nuisance parameters under the alternative hypothesis. It is interesting to review recent development in this area and link them with the work in the ACD models.

It is well established that ignoring structural breaks in financial time series can lead to false integrated models and yield long memory in the autocorrelation function (Andreou and Ghysels, 2008). Mikosch and Starica (2004) illustrate empirically that if one ignores changes in the mean or variance, the sample ACF can yield false long-range effects. Early empirical papers present the consequences of unaccounted structural breaks and regime switches in financial time series, such as Diebold (1986) and Lamoureux and Lastrapes (1990). Hillbrand (2005) presents a theoretical explanation for this false long memory effect caused by ignoring the structural changes.

Many of the conditional model structural break tests are based on GARCH models. For example, Kulperger and Yu (2005) use the partial sums of residuals from GARCH models to obtain the properties of structural break tests. Their residual-based CUSUM test shares the condition of fourth order stationarity with Giraitis, Kokoszka, and Leipus (2000) and Horvath et al. (2001). Chen et al. (2005) also use a CUSUM-based residual test based on nonparametric estimation. Andreou and Werker(2005) provide the asymptotic distribution of the CUSUM test based on the rank of GARCH residuals for detecting structural breaks, this statistic converges to a Brownian Bridge distribution and does not involve nuisance parameters. Chu (1995) and Lundberg and Teräsvirta (2002) detect structural breaks in GARCH models using the Lagrange Multiplier tests. Berkes et al. (2004) test the stability of GARCH parameters using a likelihood-ratio (LR) based test. This test is based on quasi-likelihood scores and is able to determine specific parameter's structural changes in a GARCH model. Compared with AR processes, GARCH models are more sensitive to change points in the underlying time series process. The ACD models share this property based on the strong similarity between ACD and GARCH models.

Early literature, such as Chernoff and Zacks (1964), Gardner (1969), Farley and Hinich (1970), James, James, and Siegmund (1987), Kim and Siegmund (1989), and Jandhyala and MacNeil (1991) discuss structural break tests based on a known change point. However, the change point is quite often unknown in modern financial time series, especially in the newly available intraday high frequency financial time series. Bai and Perron (1998) test multiple structural breaks at the same time, which is different from the single break at one time approach in the other papers. However, their test requires the exact number of breaks to be known within the period, and cannot be performed on conditional models. In the case of an unknown number of break points with unknown locations in conditional models, these traditional optimal tests are no longer applicable. The problem of testing multiple unknown break points is to jointly estimate the length between breaks and their locations, meanwhile providing the parameters and orders of the time series within each sub-period. When the change point is unknown, the nuisance parameter is not identified under the null hypothesis and the test statistics are not in a standard distributional form. When the time series follow a stochastic volatility process, additional difficulties arise in optimization and lead to very heavy computation. Some other empirical studies on testing multiple unknown break points include Davis et al. (2005), Lavielle and Moulines (2000), Hall and Sen (1999), and Lavielle (1999).

Recently developed work around is based on the method of binary and sequential sample segmentation. Two of the most popular structural break tests in the recent literature are the CUSUM[4] break test of Inclan and Tiao (1992), and the Lagrange Multiplier (LM) based structural break tests of Andrews (1993) and Andrews and Ploberger (1994). The CUSUM tests were initially proposed for the variance of i.i.d. processes and are widely applied to the residuals in GARCH models to test the structural breaks. The CUSUM test of Inclan and Tiao (1992) follows:

$$IT = \sqrt{T/2}\underset{k}{\max}\left[\left(\sum_{j=1}^{k} X_j / \sum_{j=1}^{T} X_j\right) - k/T\right], \tag{2.8}$$

where $0 < k < T$, $X_t = r_t^2$. The process $\{r_t\}$ is a return process which follows an ARCH (∞) process, with $r_t = u_t\sqrt{\sigma_t}$. The algorithm for detecting the variance changes is to use an iterated cumulative sums of squares. The CUSUM test is able to detect multiple breaks. However, as suggested in Smith (2008), the CUSUM test tends to reject too frequently based on the raw returns or data with fat tails, and is only able to detect breaks in the unconditional level of volatility. The CUSUM-type test has been extended in recent literature and applied to strong mixing processes (Kokoszka and Leipus, 2000).

Another popular type of structural break test for unknown location change points is the LM based tests developed by Andrews (1993) and Andrews and

[4] This test uses the iterated cumulative sums of squares algorithm to detect multiple structural changes.

Ploberger (1994). Zhang et al. (2001) apply the Andrews and Ploberger (1994) approach and divide their sample data into a number of sub-periods and hence reduce a significant level of nonlinearity effects. Three common tests of the LM based structural break tests are the Supreme (*Sup-LM*), Exponential and Weighted Average (*Exp-LM* and *Ave-LM*) tests.

The LM-based tests developed by Andrews and Ploberger are designed for testing multiple unknown breaks. Andrews (1993) solve the nuisance parameter problem by developing the sup-LM test and tabulated critical values of the test statistics. Andrews and Ploberger (1994, 1996) further extend the methods into more general conditional models. However, the LM based structural break tests failed to detect change points around the boundary of the sample data. According to Andrews (1993), the LM-based tests yield poor results if the break is too close to the boundary (they propose the initial 15% and the final 15% of the data). Andrews and Ploberger (1994) and Hansen (1996) ignore the boundary portion of data for testing structural breaks. The LM-based tests follow binary and sequential sample segmentation classes of structural break test, which is similar to the CUSUM test. The multiple change points detection applied is treated as an extension of the single change point problem. The whole sample data is first tested for the most significant structural break. If a change point is detected, the data sample is segmented into two sub-samples and retested following the same process. This process is continued until no further change points are detected. The *Ave-LM* follows

$$\lim_{c \to 0} 2(Exp\text{-}LM_{TC} - 1)/c = \int LM_T(\pi) dJ(\pi), \tag{2.9}$$

where $\pi \in (0,1)$ is the percentage location of the sample data within the change point, and c>0 is a scalar constant that depends on a weight function. If c is small, less weight is given to the alternatives for a large structural break. When $c \to 0$, the alternative hypothesis is very close to the null. In other words, the *Ave-LM* statistic is designed for alternatives which are very close to the null hypothesis by taking the limit of the Exponential LM statistics. There is also another extreme case, when $c \to \infty$, in which case the Exponential LM statistics becomes an Average Exponential form:

$$\lim_{c\to\infty} \log((1+c)^{p/2} Exp\text{-}LM_{TC}) = \log \int \exp(\frac{1}{2} LM_T(\pi)) dJ(\pi), \tag{2.10}$$

which is designed for testing against a more distant alternatives. When the constant $\frac{c}{1+c}$ is replaced by another constant $r > 0$, the limit as $r \to \infty$ of the Exp-LM statistic becomes the "*Sup-LM*" statistic. The *Sup-LM* statistics can be written as:

$$\lim_{c\to\infty} (\log Exp\text{-}LM_T^r)/r = \sup_{\pi\in\Pi^*} LM_T(\pi). \tag{2.11}$$

The *Sup-LM* test is inspired by Davies (1987). Andrews and Ploberger (1994) developed the Exponential Lagrange Multiplier statistic (*Exp-LM*) and Weighted Averages of LM tests (*Ave-LM*) to improve the power of the LM-based structural break tests. The models to which the test can be applied have also been extended from linear regression model to more general forms of models. The asymptotic uniform distribution p-values of LM based structural break tests are tabulated in Hansen (1996). Hansen (1997) investigates the restriction problems with models which contain unidentified parameters extending the work of Andrews (1993) and Andrews and Ploberger (1994). Hansen (1998) extends the LM-based structural changes tests into conditional models.

Monte Carlo experiments in Smith (2008) find that the traditional diagnostic tests such as robust LM tests for autocorrelation in Wooldridge (1990) failed to detect structural breaks in GARCH related models. According to Smith (2008), the LM-based tests in Andrews (1993) and Andrews and Ploberger (1994) are found to have more accurate size and better power to detect a range of breaks in the dynamics of conditional volatility.

Some other forms of structural break tests have also been developed; for example the Generalized fluctuation test framework developed by Kuan and Hornik (1995) and Leisch et al. (2000). Seigmund (1970), and Davis et al. (2005) present a method of testing structural breaks in a stochastic volatility process based on minimum description length criterion. There is also recent evidence which suggests that stock market volatility is better measured with short-memory processes with level shifts. Granger and Hyung (2004) show that long memory parameters with regimes are significantly reduced by allowing structural breaks in the process. Beltratti and Morana (2006) apply structural breaks to exchange rate returns and yields very good forecast results. Structural break tests are also applied in the tails of distribution of time series, such as Quintos et al. (2001) and Mickosch and Starica (2000). The structure of the financial returns' distribution function can also be tested for structural change. Some of the non-parametric change point tests for distribution function changes can be found in Inoue (2001) and Lavielle (1999). Sowell (1996) also extend the tests for parameter instability to the GMM framework.

2.7 Summary

In this chapter, a review of duration modelling is presented. The review started with the necessity of studying durations and their close relation with market microstructure theory. The original ACD model from Engle and Russell (1998) and some of its extensions are also reviewed. Types of tests for testing the ACD models and types of durations have been applied are then discussed. In addition, literature based on structural break study is also presented.

In the following chapter, we introduce the after-hours electronic futures market and its backgrounds and developments. The background, build up, and consequences of the global financial crisis are also introduced.

Chapter 3

The After-Hours Electronic Market and the Global Financial Crisis

3.1 Introduction

As the duration modelling in this thesis is based on the after-hours electronic market, section 3.2 provides an introduction on developments, brief history, and operations of this market. Since most electronic futures contracts are transacted through GLOBEX, this electronic platform is also introduced. An overview of the 2007-2008 global financial crisis is included in section 3.3, as in the final part of the thesis the sample period extends to include these years.

3.2 The After-hours Electronic Market

3.2.1 Overview

Over the last decade, an increasing volume of trading activity has moved towards electronic exchanges and trading hours have extended beyond standard business hours. Consequently the volume in these electronic markets has grown tremendously, and currently the electronic markets are in a dominant position in the trading institutions. For example, the statistics from the Chicago Mercantile Exchange (CME) group shows that during 2008, about 80% of the volume traded was transacted electronically. With the development of improved technology, electronic trading attracts increasingly more participants and plays an important role in the current trading era. Interestingly there have only been few empirical studies on this after-hours electronic market. This overview of this market contributes in building up the duration modelling of this thesis. The overview begins with the developments in electronic market in the following section.

3.2.2 Developments in Electronic Markets

In a floor (open outcry) market, physical locations and human interventions are needed to gather buyers and sellers to negotiate for transactions. With developments in computer technology, the need for physical location becomes less important. According to Levecq and Weber (2002), following NASDAQ, the first electronic stock market set up in 1970s, more exchanges have sought to expand their trading services through electronic trading. Electronic trading can be more convenient and can be accessed in remote locations and over extended trading hours.

Nowadays the electronic trading offers more convenient trading and easier access than the floor market. Most of these electronic markets make the limit order book visible to investors and apply a standard price/time priority structure to ensure the flow of transactions. When new bids (offers) are submitted to market, they are assigned priority from the highest (lowest) price in the limit order book. Orders with equal prices are then processed according to the time of entry. In electronic markets, investors must enter their orders into the system and any modification or cancellation requires another transaction. Levecq and Weber (2002) argue that the physical entry and the response time from the platform for a modification or cancellation can create a delay. This delay between the traders' intentions and intentions listed on the market exposes traders to risk in receiving unwanted executions or missing a trade. For this reason and many others, the floor based market still owns a certain portion of participants. Currently electronic contracts trade simultaneously with the floor, after-hours electronic trading.

Some of fully automated electronic markets include the German Deutsche Terminborse (DTB), the Swiss Options and Financial Futures Exchange (SOFFEX), the Irish Futures and Options Exchange (IFOX) and the New Zealand Futures and Options Exchanges (NZX). The most popular electronic trading is the CME Group's Global Exchange platform, GLOBEX, which covers financial, foreign exchange, equity, commodity and many more markets. Details on GLOBEX are presented in the next subsection. A number of other electronic markets now also offer after-hours trading systems, such as American Computerized Commodity Exchange System and Services (ACCESS) from the New York Mercantile Exchange; Sydney Computerised Overnight Market (SYCOM) in Australia; the

French Marché à Terme International de France (MATIF), and the Automated Pit Trading market (APT) from the London International Financial Futures Exchange.

3.2.3 GLOBEX

The GLOBEX electronic trading platform has become the world's largest electronic futures exchange, offering the broadest varieties of derivative products around the clock and around the globe. The GLOBEX platform is investigated in detail in the following subsections.

History

According to the CME group web site[1], the GLOBEX platform was originally proposed by Chicago Board of Trade (COBT) for the purpose of allowing evening floor trading in the late 1980s. They soon found the demand for trading from non-U.S. trading hours overwhelming. In 1987 the CME Group passed the proposal and the first global electronic platform for futures contracts was launched in 1992. The percentage of GLOBEX electronic trade volume was only 1% of the whole market in the year 1996 and just under 10% before 2000. The CME group also expanded the varieties of trading products offered.

An important milestone "open access" policy was implement in 2000. Since then the interest in the electronic markets has grown steadily. Customers are able to trade directly, with a financial guarantee from their brokers on GLOBEX, without the obligation to route orders through a broker via telephone This policy lead

[1] Relevant websites include: http://www.cmegroup.com/globex/developin g-to-cme-globex and http://www.cmegroup.com/globex/resources/history-of-globex.html.

to a huge increase in the volume traded on the electronic market. More traders switched over from the pit market. In 2002, the average daily volume transacted electronically in GLOBEX exceeded 1,000,000 contracts. The GLOBEX volume exceeded pit market volume in 2004 and after that the GLOBEX began to take over market in term of total volume traded. Later in the same year a record 1 billionth transaction took place. By the end of 2004 the GLOBEX exchange accounted for 58% of all trading volume within all the trading markets in the CME group.

In 2007, CBOT was merged with the CME group, and in that year GLOBEX volume exceeded 1 billion contracts with the additional volumes acquired from CBOT. In August 2008, the CME group's market position was further improved through the acquisition of the New York Mercantile Exchange (NYMEX) (and its sub-division COMEX[2]). Consequently the CME group now includes the CME, CBOT, NYMEX, and COMEX, and has become the largest futures and options exchanges in the world, offering the widest range of global benchmark products across all major asset classes. The CME group electronic markets are also one of the most liquid in the world. From the latest information available[3], up to 2008, the volume traded in GLOBEX reached a historical record of 82% of the market. The CME continues to introduce new contracts such as the E-micro Foreign Exchange Contracts[4] and the new ASCI products[5].

The GLOBEX volume percentage of the market was reduced in 2009-2010 due to the global financial crisis, but still finished up with 80% of the market

[2] The statistics are collected from CME group Annual Reports from 2002 to 2009.

[3] From 2009 GME group Annual Report.

[4] These contracts are 1/10th the size of the corresponding forex futures contracts, making trading more accessible to a wider range of investors.

[5] Argus Sour Crude Index (ASCI), it reflects the value of sour crude oil traded and delivered in the U.S. Gulf Coast market.

transaction volumes. The open pit market still holds around 20% of the market volume. As argued by Coppejans and Domowitz (1999), floor markets have a unique traditional way of trading and will continue to coexist with the electronic market due to the structures of fee and charges, margins, and other factors. In 2008, there were more than 1,100 direct connections to GLOBEX in more than 86 countries and foreign territories[6]. There are also telecommunication hubs in Singapore, London, Amsterdam, Dublin, Milan and Paris to ensure faster and more efficient trading with lower connectivity costs.

Operation Cycle and Clearing

The GLOBEX order routing interface is supported by iLink, which is connected based on the FIX 4X protocol. It allows traders to access the electronic markets whenever they are open and supports customized trading systems to enter, modify and cancel orders and receive order confirmations. Anyone who has an account with a Futures Commission Merchant (FCM) or an Introducing Broker (IB) who has a CME Clearing Guarantee, can trade on the platform. Customers from all over the world are able to participate trading through GLOBEX platform, enter orders and view book of orders and prices of CME group products directly. One restriction of this platform is that it only offers limit orders[7].

Generally there are five entry states through the GLOBEX futures market daily trading cycle, namely the enabled state and no-cancel state in the pre-opening period, continuous trading state, surveillance intervention state, and system maintenance state. The CME GLOBEX session starts at a predetermined time before the trading session opens. In a market enabled/pre-opening state,

[6] These statistics are collected from CME Globex Reference Guide.
[7] This restricts the broker to a given price/or better in bidding/offering.

traders can enter, modify and cancel orders for the next trading day; in a pre-opening/no-cancel state traders can only enter orders for the next trading day without modification or cancellation. When the trading session actually starts, it enters the continuous trading state. In this period of time orders are sent and matched in real time. When the trading session closes, there is a surveillance intervention state. In this state traders can only process cancellations. The last state is the maintenance period, no order entry, modification or cancellations can take place during this time. In cases of emergency, traders may still cancel orders through the GLOBEX Control Centre[8] (GCC).

As a combination of the CME, CBOT, and NYMEX, the CME group has its own Clearing House, which guarantees and processes all matched transactions of CME group contracts and ensures the transactions' financial integrity. Currently the CME Clearing House is one of the largest organizations for clearing in the world and it handles approximately 90% of all U.S. futures and options on futures volume. It monitors and settles more than one billion trades every year, worth more than $1,000 trillion.

Products Range

The products currently covered on GLOBEX include equity, interest rate, foreign exchange, commodity, real estate, weather, the NYMEX and COMEX products, Kansas City Board of Trade (KCBT) and Minneapolis Grain Exchange (MGEX) products, Total Return Asset Contracts (TRAKRS[9]), and OneChicago Security futures products. Other well known indexes such as S&P MidCap 400, S&P

[8] See CME Globex Reference Guide at http://www.cmegroup.com/globex/files/GlobexRefGd.pdf for more detail

[9] The first broad-based index products traded on a U.S. futures exchange to be sold by securities brokers. See more details on www.cmegroup.com/equities.

SmallCap 600, Dow Jones Industrial Average, Nikkei 225 Stock Average are also traded. Not all electronic trading on CME GLOBEX are after-hours. Some contracts trade simultaneously with the floor in side-by-side market (Foreign Exchange), and some contracts are only traded electronically (E-minis and E-micros). The S&P 500 and NASDAQ-100 data used in the thesis are from the equity product range. In the following, the standard S&P 500 and NASDAQ-100 contracts, E-minis, and E-micros are introduced.

S&P 500 and NASDAQ-100 The data set used in this thesis is the standard NASDAQ-100 and S&P 500 equity futures contracts traded on GLOBEX. The S&P 500 futures contract was launched in GLOBEX on the 1995 and NASDAQ-100[10] in 1996. The standard contracts that trade in after-hours time are the same contracts which trade in the pit market during the day trading hours. The platform closes for maintenance everyday between 16:30 (All times are in CST) and 17:00. The trading hours for most of the electronic contracts are 17:00 to 8:15 and 15:30 to 16:30 with the exception of Fridays where there is no electronic trade following the closure of the open outcry pit on Friday afternoon. On Sundays trading begins at 17:00 and finishes at 8:15 on Monday morning. On public holidays the platform trades reduced hours. The product fees in open outcry and GLOBEX can be different and charges can be higher or lower in two markets. Fees are also charged per side (buy and sell) per contract. A clearing fees of $12.5 is required for both markets.

The standard equity futures contracts for the NASDAQ-100 traded on the CME are contracts for $100 multiplier times the equity index price with 0.25

[10]Nasdaq-100 includes the top 100 large-cap domestic and international non-financial companies on the Nasdaq Stock Market.

ticks. For the S&P 500, the multiplier is $250 and minimum tick is 0.10. The trading of the same products do not overlap between the open outcry pit and the electronic trading, but it is possible for traders to change their portfolio holdings in these indices almost 24 hours a day by using both electronic and pit markets.

E-minis E-minis offer one-fifth the size of a standard contract, with these smaller contracts designed to appeal to retail investors. It is available electronically and trades 24 hours other than during GLOBEX maintenance hours. The E-mini versions of S&P 500 and NASDAQ futures contract were launched in 1997 and have become very popular since early 2000. At that time the existing standard contract became too large for many small traders. The E-minis contract has quickly became the most popular equity index futures contract in the world. Hedge funds preferred the E-minis in early 2000, when the majority of standard futures contracts were still mostly traded in open outcry market[11].

E-micros The E-micro was introduced by the CME in 2009. It is a smaller version (one-tenth) of standard size foreign exchange futures contracts. The E-micro foreign exchange includes 6 currency pairs, namely EUR/USD, USD/JPY, GBP/USD, AUD/USD, USD/CHF and USD/CAD. It aims to attract more retail investors and provides reduced margin requirements. In 2010, E-micros started to offer E-micro S&P CNX Nifty futures contracts in equity class[12] The risk exposed in the E-micro is also smaller and it has becoming a more regulated marketplace. All contracts including standard sized futures, E-minis, and E-micros are settled by CME Clearing.

[11] See Growth of CME Globex Platform available at http://www.cmegroup.com.

[12] It is offered as International index futures contracts.

3.3 Overview of the Global Financial Crisis

3.3.1 Introduction

The 2007-2008 Global Financial Crisis (GFC) is generally regarded as the worst financial crisis since the 1930s. It has resulted in losses in many leading banks and large financial institutions due to the collapse in value of mortgage-based securities in the US and Europe. This period of duration data may contain rich source for studying potential structural changes of the after-hours market. Moreover, the approach of studying the global financial crisis through trade duration is unexplored in the GFC literature. As chapter 7 of this thesis tests for structural breaks in ACD models around this financial crisis period of data, it is necessary to review the events of the crisis period. In the following subsections, a brief introduction to the background, causes, and consequences of the subprime financial crisis is presented.

3.3.2 Prior to the Crisis

Following the dot-com collapse in 2001, the US Federal Reserve kept interest rates very low due to fears of recession. To keep the national consumption level and US consumer's purchasing power continually rising became a national security priority following the terrorist attack in 2001 (Blackburn, 2008). Under this policy, historically low interest rates made loans cheap and easy. The US Federal Reserve created easy credit conditions and encouraged debt-financed consumption.

Consequently debt related investments grew significantly in the early 2000s. According to the statistics shown in Blackburn (2008), the total debt in the U.S. economy increased from 255.3% in GDP in 1997 to 352.6% in GDP in 2007. These huge increases were particularly strong in the household and financial sectors, such as banks and other financial mortgage institutions. The number of mortgage-backed securities (MBS) and collateralized debt obligations (CDO) issued also increased. Investments from all around the world came to take advantage of this policy and heavily invested in the mortgage backed market, especially the housing industry. As a consequence of increased demand, housing prices steadily increased throughout 2001 to 2006.

On the other hand, the banking system was tainted by poor systematic risk management. The so-called shadow banking system[13], which consists of non-depository banks and financial entities including investment banks and hedge funds, developed dramatically over the same period. Some of the major examples include Citigroup, Merrill Lynch, HSBC, Barclays Capital and Deutsche Bank. Under this shadow banking system, entities create huge leverages by borrowing extra debts from investment banks and hedge funds. In some entities, their borrowed assets were worth almost thirty times of their capital. After the early 2000s, this shadow banking system quickly expanded as the rules that govern borrowing and lending were loosened. This huge portion of debt created significant levels of potential systematic risks in the market. As many of these debt-mortgages were multi-nationally related, huge leverage was formed by these debt-related assets in the banking and mortgage institutions. Goodhart (2008) pointed out that the low interest rates lead to under-pricing of risk and also low risk spreads, thus the liquidity of banks decreased dramatically. The reduced liquidity implies that if there is a problem in the banking system, the central

[13] Financial intermediaries involved in this system create credit across the globe and its members are not subject to regulatory oversight. Some of these intermediaries include hedge funds, unlisted derivatives and other unlisted instruments. The shadow banking system did not accept traditional bank deposits, therefore it has escaped from regulation.

bank is the only one that can be relied on by the lending banks. While US housing prices continued to rise, banks encouraged considerably higher loans. This bubble peaked around 2005 and 2006.

3.3.3 The Prelude

Throughout 2005 to 2006, US interest rates began to rise towards more historically normal levels due to the pressure of a falling US dollar. This rise in interest rates caused fears in lending markets, which were especially obvious in the housing industry. As a result, US housing prices, which had been growing steadily, started to decline. Increased interest rates caused rising defaults in mortgages and meanwhile, shrinking housing values began to cause losses in the lower tranches of CDOs and collateral mortgage obligations (CMO). Institutions holding these instruments, such as hedge funds started to suffer losses. Coupled with the fear of defaults hitting the higher tranches, the market turned bearish through late 2006 to early 2007.

The increasing fear of defaults created a credit crunch in mid 2007, investors started to lose confidence in the value of sub-prime mortgages. Leading investment banks in US and Europe were also affected since their asset base constituted a large portion of U.S. mortgage based securities. According to Pezzuto (2008), investment banks started to be suspicious about the potential credit losses of their counterparties which led to a tightening inter-bank lending. As banks reduced their exposure in the interbank markets the inter-bank lending interest rates rose. This also led to increased rates for credit default swaps (CDS), making the liquidity problems in the banking system and credit crunch conditions even worse.

3.3.4 The Crisis

It is commonly accepted that the global financial crisis started in July 2007, followed by the collapse of Deutsche Bank and failure of large mortgage brokers. By the early August 2007, the world's central banks were injecting liquidity into the global financial system but were still unable to stop the crisis from building. Investors lost confidence and tightened their spending in fear of further losses. In September 2008, the Lehman Brothers collapsed and filed for bankruptcy protection. Merrill Lynch was also sold to the Bank of America in the same month. Soon credit ratings started to decline for American International Group (AIG) and the US Federal Reserve had to lend $85 billion to AIG to avoid bankruptcy. Meanwhile the US Federal Reserve continued to inject liquidity into the market, and the US government carried out a series of plans to calm the market. However, the crisis continued to spread throughout US and worldwide.

3.3.5 The Aftermath

As a consequence of the GFC, many financial institutions continue to face liquidity issues. Following worsened housing and stock markets, governments around the globe struggled to save large financial institutions. These collapses of large US financial institutions and downturns in stock markets quickly spread out to the global financial market.

By November 2008, the US stock index S&P 500 had dropped around 20% from its peak level in 2007. Housing prices had also dropped 20% from their 2006 high[14]. Altman (2009) suggests that the total home equity value had also shrunk

[14]Data collected from Yahoo finance.

dramatically from its peak level of \$13 trillion in 2006 to \$8.8 trillion in mid-2008. By the end of 2009, the International Monetary Fund (IMF) estimated that large banks in US and Europe lost more than \$1 trillion.

Since modern financial markets are highly integrated at a global level, the sudden rise in US financial market volatility and risk aversion was quickly transferred worldwide. The sharp drop in demand for capital intensive goods was also quickly transmitted through the global supply chain. In the UK, the rising cost of liquidity caused losses in mortgage house related financial entities, and also lead to the first bank run in the UK for 150 years at Northern Rock. A number of other European banks also suffered. Stock indices and the market value of equities and commodities in Europe also declined. The after-hours electronic market during global financial crisis period is investigated in chapter 7 of this thesis.

3.4 Summary

This chapter provides backgrounds and overviews of the after-hours electronic market and the 2007-2008 global financial crisis. Trading volume in the after-hours market has grown significantly in the past years, of which the GLOBEX in CME group is the most liquid trading platform in the world, offering the widest varieties of products. The history of GLOBEX developments is reviewed in order to gain a better understanding of the market. Information on products available in this market is presented to improve the understanding of trading mechanisms, including some of the popular futures products such as E-minis and E-micros. The second section reviews the build-up of the global financial crisis. The large volume of duration data during the crisis provides opportunities to explore duration data linear and nonlinear behaviours. Considering the data sample in this thesis is relatively long (2 years), it provides an unique empirical study on duration modelling under potential structural changes. Some of major economic events during the crisis are also addressed to back up the structural analysis in chapter 7.

The following chapters include four papers. In chapter 4, the empirical analysis part of this thesis begins by fitting simple linear ACD specifications. Then the analysis moves towards more complicated nonlinear model in chapters 5 to 7. Two types of nonlinear forms of ACD models are addressed based on after-hours market in this thesis, namely the logarithmic ACD model in chapter 5, and ACD model with structural breaks in chapter 6 and 7.

Chapter 4

Linear ACD models

4.1 Introduction

This chapter[1] studies the time between trades of the after-hours electronically traded equity futures market, a market which is previously unstudied in this regard. Using a relatively long (2 years) data set, trades in the NASDAQ and S&P 500 equity futures are shown to require different forms of autoregressive conditional duration models, including longer lag lengths than previous spot data applications. Additionally, volume provides an informative mark in both cases. The S&P 500 necessitates a threshold model where the majority of trades display the typical low autocorrelation and strong clustering evident in other assets, but with large durations more autocorrelated with weak clustering.

The trading environment in financial markets has changed rapidly in the past 10 years. Many instruments are increasingly traded on electronic exchanges and trading hours are extending beyond standard business hours. A particularly successful example of these innovations is the trade in equity futures contracts on

[1] This chapter consists of joint work with Mardi Dungey and Nagaratnam Jeyasreedharan, as stated in the Statement of Co-Authorship enclosed with this thesis.

the GLOBEX exchange. Equity futures contracts which trade on the open outcry Chicago Mercantile Exchange (CME) pit are now also generally available outside pit hours on the electronic market. Since 1993 the standard size contract for the S&P 500 has been available in this format, followed in the mid-1990s by the NASDAQ contract, and growth in volume has been relatively rapid. However, to date, the behaviour of the after-hours market has been relatively little studied; Coppejans and Domowitz (1999) compare the electronic and open outcry markets and Dungey, Fakhrutdinova and Goodhart (2009) explore the volume and volatility characteristics of the NASDAQ and S&P 500 futures contracts.

This chapter makes three contributions. First, it considers trade duration, that is the time between trades, in the after-hours equity futures markets for the NASDAQ and S&P 500 indices. The time between trades provides information to the market, indicating the presence of news and potentially in the absence of trade that there is no new information, see Easley and O'Hara (1992). Trade duration has not previously been modelled for the after-hours market. Being after-hours it has a peculiarly marked diurnal pattern, with relatively intense trade in the period immediately following the close of the open outcry market, lower volume and intensity in the Asian trading zone, an increase in activity and intensity in European trading hours and a dramatic increase in both intensity and volume immediately prior to the opening of the pit - corresponding particularly with the 8:30am EST scheduled macroeconomic news announcement period in the US. Modelling trade duration in this market is thus a completely different proposition from previous empirical work on duration modelling, which typically involves spot equity market contracts; for example Engle and Russell (1998), Zhang, Russell and Tsay (2001).

Second, the data sample of this chapter covers two years, a significant increase on the usual 3 month sample analyzed in existing papers on time between trades. A particular challenge is to fit a consistent model to this length of sample - given that Zhang, Russell and Tsay (2001) find evidence for 7 structural breaks in a 3 month data set. The final contribution is to include volume of trade as an additional mark in the modelling process, which makes a small, but significant, negative contribution to conditional duration. That is, an observed larger trade volume results in a smaller time to the next trade - which may be interpreted as either due to the arrival of public information resulting in market participants making portfolio adjustments, or alternatively in the absence of public information, that when market participants observe a high volume trade they interpret this as private information which encourages them to trade, thus increasing trade intensity.

The modelling framework of the chapter is based on the ACD models proposed by Engle and Russell (1998) and subsequent extensions. The ACD models account specifically for the observed serial correlation and clustering in trade duration, and are closely related in form to the GARCH framework. Like GARCH, the preferred lag structure in most applications strongly suggests an ACD(1,1) starting point, although various alternatives exist for the assumed error distribution; including the Exponential, Weibull, Generalized Gamma, Burr, Generalized F and mixtures of distributions; see Russell and Engle (1998), Lunde (2000), Grammig and Maurer (2000), Hautsch (2002) and De Luca and Gallo (2004). The markets explored here require both an extension of the lag structure and accounting for non-linearities through a two regime threshold ACD model. Specifically, the duration model on the NASDAQ futures data incorporates higher order lags, while the more intensely traded S&P 500 contract is more effectively modelled with a threshold model, featuring differing levels of higher order lags for large duration observations.

The chapter proceeds as follows. Section 4.2 provides a brief overview of the after-hours electronic equity futures market for the NASDAQ and S&P 500 contracts, followed by the description of the sample period in Section 4.3. The ACD framework is outlined in Section 4.4. Section 4.5 documents the development of the final model via the benchmark ACD(1,1) model, extensions to the lag order, the introduction of volume and threshold models. Section 4.6 concludes.

4.2 The After-hours Electronic Equity Futures Market

The standard equity futures for the NASDAQ and S&P 500 traded on the CME are contracts for $250 times the equity index price with 0.10 ticks. Both contracts trade in the CME open outcry pit between the hours of 8:30 CST to 15:15 CST and on the electronic GLOBEX exchange after-hours. The after-hours trading period currently begins at 17:00 CST on Sunday evenings (corresponding to the opening of trade in the Japanese trading day) and continues until 8:15 CST Monday morning. For the remainder of the working week the contract begins trade at 15:30 CST after the closure of the pit, and continues to trade until 8:15 CST the next morning, with the exception of Fridays where there is no electronic trade following the closure of the open outcry pit on Friday afternoon. The electronic exchange closes for maintenance everyday between 16:30 CST and 17:00 CST, and on public holidays trades reduced hours.

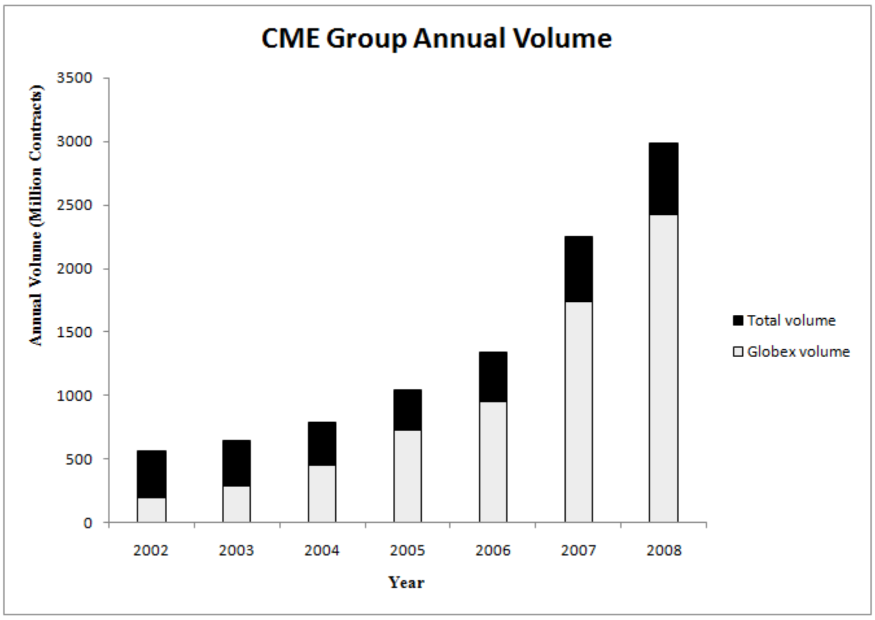

Figure 4.1: Annual Volume of Trade on the CME

There is no overlap in trade of the open outcry pit and the electronic trading of these contracts. The two platforms are trading the same product, thus making it possible for market participants to change their portfolio holdings in these indices almost 24 hours per day. Although there is no electronic trading in the standard contract during the open-outcry market, E-mini contracts which are one-fifth of the standard contract size and only available electronically do trade 24 hours (other than the half-hour shutdown for maintenance).

Total volume accounted for by electronic trade in this market has been growing rapidly in recent years. Figure 4.1 shows that total volume traded in the electronic market has grown from 200 million in 2002 to more than 2 billion 2007, although this includes the consolidation of the CME and CBOT trades into the total volume in 2007.

It is not at first evident how 4 contract forms (standard future, electronic, E-mini, and E-micros) for the same instrument coexist. However, the standard contract trades electronically only when the pit floor is closed and is five times larger than the E-mini product which trades for virtually 24 hours. The E-micro version of S&P CNX Nifty only became available in 2010 and the E-micro contracts are still more developed in foreign exchange products. The smaller contract is designed to appeal to retail investors. Additionally, trade on the electronic platforms is more expensive than trade in the pit via transaction fees, although precise details of the transaction fees vary by market participant and are not readily and publicly available. As the E-minis trade during the pit period of 8:30 CST to 15:15 CST. Hasbrouck (2003) and Coppejans and Domowitz (1999) have compared the relative efficiency of the E-mini and open outcry market - finding that the open outcry market is more efficient at absorbing local information. However, this comparison is made more difficult by the difference in size and transaction fees of the contracts. Trading in the pit and on the electronic platform for the standard contract do not overlap - rather in combination they complete the trading day, so their relative efficiency can not be easily compared.

Dungey, Fakhrutdinova and Goodhart (2009) investigate volume and price impact for the after-hours standard equity futures contracts for the S&P 500 and NASDAQ indices. They find that the period of highest average volume in the day occurs immediately prior to the opening of the open outcry pit, peaking around 7:30 CST, which corresponds to the time of prescheduled macroeconomic news releases in the US at 8:30 EST. They find that price impact for the S&P 500 contracts is lowest in the high volume period immediately prior to the opening of the open outcry pit, and higher in general during the European and Asian trading hours, but for the NASDAQ, price impact is highest immediately post-close of the open outcry market. This may suggest that the relatively low volume traded on the NASDAQ compared with the S&P 500 has made the gains from anonymous electronic trading lower than those for the highly liquid S&P 500, reducing the attractiveness of trade in the post-close period for this instrument.

4.3 The Data Sample

Information on the transactions on the GLOBEX electronic exchange for the NASDAQ and S&P 500 futures contracts were obtained from the CME for the period from July 1, 2004 to September 30, 2006. The data comprise 213,332 observations for the NASDAQ and 1,053,524 observations for the S&P 500. Following Engle and Russell (1998), after cleaning the data set to remove negative durations[2] and aggregating volume for transactions with the same time stamp to be treated as a single transaction, the sample data was found to contain 149,314 observations for the NASDAQ and 684,010 observations for the S&P 500. The data display a distinct diurnal pattern, and it is customary in this literature to remove this pattern prior to estimation. Using a piecewise linear spline with 17 knots representing hourly intervals during the after-hours trade period covered the data are diurnalised using a multiplicative specification of the diurnality, in a manner similar to that proposed in Engle and Russell (1998).

Table 4.1 contains basic descriptive statistics of the diurnally adjusted duration and volume, clearly indicating the near unit mean. Variance of the adjusted durations for NASDAQ is slightly higher than S&P500, suggesting larger fluctuations on the waiting time and less frequent trading activities in NASDAQ in our sample data. The lower variance of the adjusted volumes for NASDAQ further implies a less volatile market. In both indices there is evidence of relatively large

[2] Negative duration is possible here since the trading time continues over mid-night.

Table 4.1: Descriptive Statistics
for Adjusted Durations and Adjusted Volume
in the NASDAQ and S&P 500.

	NASDAQ	S&P 500
Duration		
number of observations	149314	684010
mean	0.9994	0.9973
max	66.3147	70.5960
min	0.0014	0.0061
variance	4.1081	3.6010
skewness	5.7514	5.4368
kurtosis	67.4259	62.4087
Jaque-Bera (p-value)	0.0000	0.0000
Volume		
mean	1.0000	0.9999
max	67.1146	116.9578
min	0.2262	0.2102
variance	2.0121	2.3732
skewness	6.8145	9.4208
kurtosis	117.2487	270.6885
Jaque-Bera (p-value)	0.0000	0.0000

higher order moments, strongly rejecting normality. Figures 4.2 and 4.3 show the average adjusted daily duration and volume pattern for the NASDAQ and S&P 500 data beginning from midnight CST each day. Trade at midnight CST is equivalent to the Asian trading day, and the durations are relatively high. Duration then decreases until 8:15 CST when the GLOBEX market ceases shortly before the open of the pit trading session. During the morning electronic trade duration drops first during the European trading day and most dramatically around the 7:30 CST period (corresponding to the usual announcement time for pre scheduled US macroeconomic news). As discussed in the previous section, diurnal volume in these markets peaks at this time.

Immediately following the closure of the floor market at 15:15 CST trading is relatively intense in the electronic market, and volume is again relatively high. Dungey, Fakhrutdinova and Goodhart (2009) associate this higher trading volume

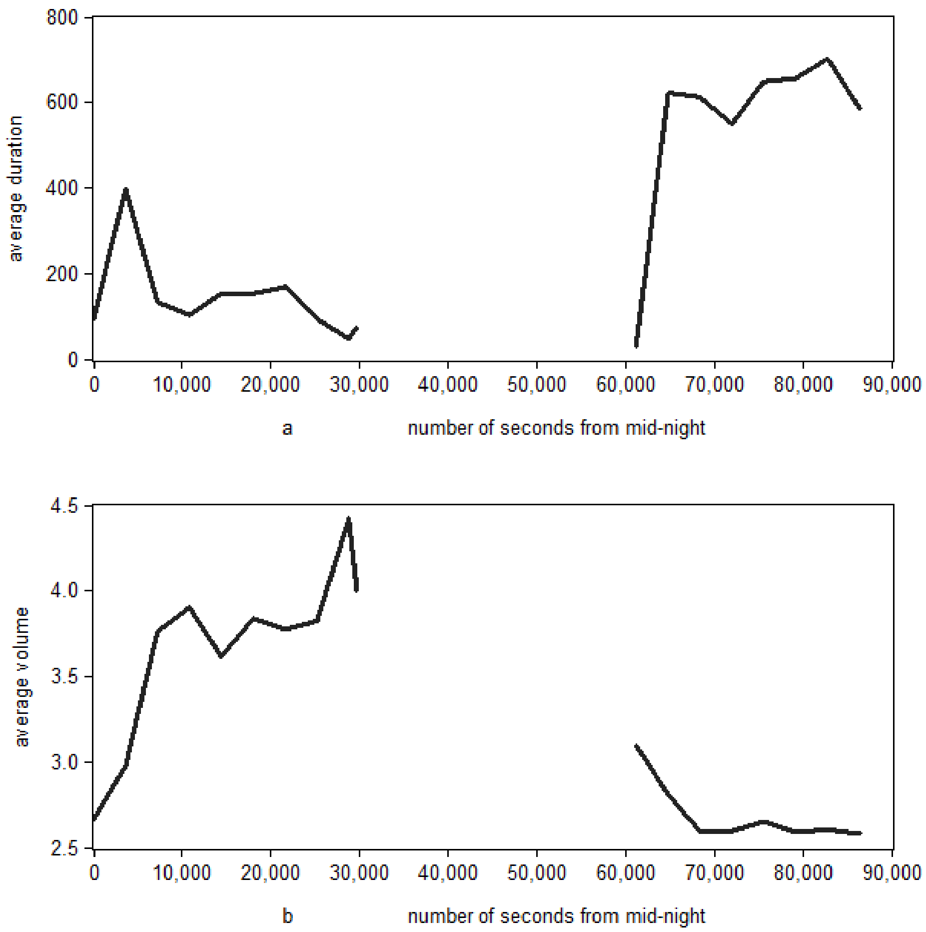

Figure 4.2: Diurnal (a) Durations and (b) Volume Patterns for NASDAQ

with a desire on the part of market participants to settle their end of day positions in the anonymity of the electronic market as opposed to the open outcry pit, despite the higher costs of trading the same contract on the electronic market. After this point, trade duration begins to climb again as the market becomes less active entering the Asian trading zone. Overall, the figures indicate the existence of a negative relationship between volume and duration. This feature will be incorporated into the formal model of duration in Section 4.5.4.

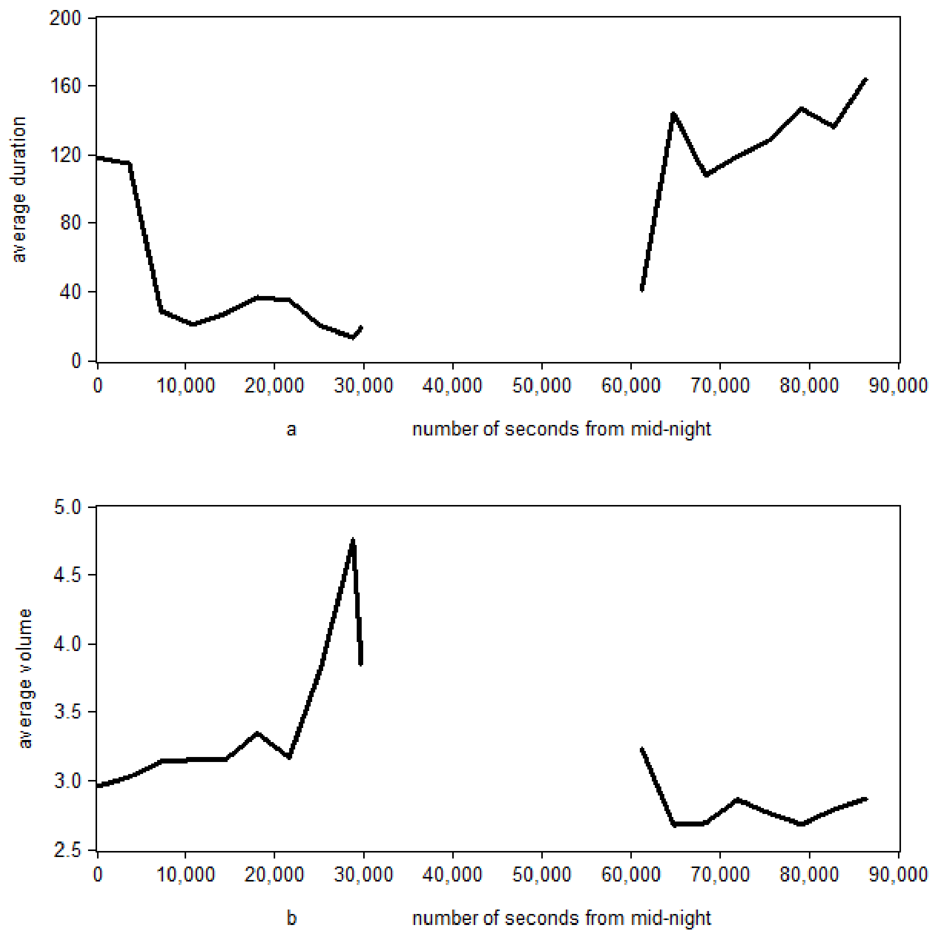

Figure 4.3: Diurnal (a) Duration and (b) Volume Patterns for S&P 500

4.4 ACD Models

The (irregular) time between consecutive trades in a single market is defined as $x_i = t_i - t_{i-1}$, where t_i represents the time of the current trade and t_{i-1} is the immediately previous trade. Assuming that the trade duration, x_i, evolves according to the process

$$x_i = \psi_i \varepsilon_i, \tag{4.1}$$

where $\psi_i \equiv E(x_i | x_{i-1}, \ldots, x_0)$ represents conditional expected duration and ε_i is an error process, the autoregressive and clustering aspects of duration are captured through specification of the conditional expected duration as

$$\psi_i = \omega + \sum_{j=0}^{p} \gamma_j x_{i-j} + \sum_{k=0}^{q} \omega_k \psi_{i-k}, \tag{4.2}$$

where ω, γ_j and ω_k are parameters, and p and q represent the lag orders, denoted as an ACD(p, q), see Engle and Russell (1998).

A number of alternatives have been considered for the error distribution ε_i, including the Exponential (EACD), Weibull (WACD), Generalized Gamma distribution (GGACD), Burr and Generalized F; see Engle and Russell (1998), Lunde (2000), Grammig and Maurer (2000), and Hautsch (2002). De Luca and Gallo (2004) use a mixture of two distributions.

This chapter concentrates on comparisons of the EACD, WACD and GGACD forms of the model. In each case the duration, x_i, is restricted to be non-negative The probability density function

$$f(x) = \frac{\alpha}{\beta^{\alpha}\Gamma(\kappa)} x^{\kappa\alpha-1} e^{(-x/\beta)^{\alpha}}, \tag{4.3}$$

represents the Generalized Gamma distribution with two shape parameters, α and κ and scale parameter β, which in the case of $\kappa = 1$ is equivalent to the Weibull distribution and when $\alpha = \kappa = 1$ is the Exponential distribution. Each of these functions possesses high concentration at shorter durations and a long right tail for longer durations.

A number of alternative specifications to the conditional duration given in equation (4.2) also exist. Expressing equation (4.2) in log form rules out negative durations which have occurred in other applications with the addition of further explanatory variables to the conditional duration model; Bauwens and Giot (2000), but are not an issue in the current application. Jasiak (1998) introduced the fractionally integrated ACD model, the FIACD to account for long memory, while Zhang, Russell and Tsay (2001) introduced the threshold ACD model, where different conditional means, error distributions and persistence are allowable in each regime. In the two regime threshold model, the conditional duration equation (4.2) is replaced by

$$\psi_i = \begin{cases} \omega^{(1)} + \sum_{j=1}^{p_1} \gamma_j^{(1)} x_{i-j} + \sum_{k=1}^{q_1} \omega_k^{(1)} \psi_{i-k}, \text{ if } 0 < x_i \leq r_1 \\ \omega^{(2)} + \sum_{j=1}^{p_2} \gamma_j^{(2)} x_{i-j} + \sum_{k=1}^{q_2} \omega_k^{(2)} \psi_{i-k}, \text{ if } r_1 < x_i < \infty \end{cases} \tag{4.4}$$

which is notated as TACD$(p_1, q_1 : p_{2,}q_2)$ where p_1 and q_1 represent lag orders in the first regime, and p_2, q_2 represent lag orders in the second regime and r_1 is some exogenously chosen cut off point delineating the regimes. Other recent alternatives include Markov Switching ACD models, as in Hujer et al. (2003); mixtures of distributions applied to price durations in De Luca and Gallo (2004) and trade durations in Hujer and Vuletić (2007), stochastic volatility duration models such as Ghysels, Gourieroux and Jasiak (2004) and the simultaneous modelling of price and trade duration in Engle and Russell (2005).

The next section presents the results of applying the ACD model with varying error assumptions and threshold ACD specifications to the NASDAQ and S&P 500 equities futures data. Parameter estimates are undertaken using maximum likelihood based on the log-likelihood functions for the individual models using RATS version 7.

4.5 Empirical Results

The majority of the existing literature has fitted ACD$(1,1)$ models with alternative distributional assumptions. EACD(1,1), WACD(1,1) and GGACD(1,1) models are fitted to the two data series in the next section, followed by extensions to higher lag orders and then the potential role of volume traded in providing further information. Finally, evidence of non-linearity in the S&P 500 results lead to the estimation of a threshold ACD model for this data.

4.5.1 ACD(1, 1) Specifications

Table 4.2 reports the coefficient estimates, Ljung-Box statistics and AIC and SBC statistics for EACD(1,1), WACD(1,1) and GGACD(1,1) models for the NASDAQ and S&P 500 data. Consider first the results for the NASDAQ data reported in Table 4.2. The Ljung-Box statistics for each model are relatively high, ranging between 280 and 372 for the Q(20) statistic , although this reflects the large sample size in addition to potential problems with the fit of the model. The parameter estimates in the GGACD(1,1) and WACD(1,1) also provide some evidence as to which model best describes the data. There is considerably more variation in the parameter estimates for autocorrelation and clustering across the specifications than obtained by De Luca and Gallo (2004) in their comparison of ACD(1,1) models for price durations across different distributional assumptions. The parameter estimates for κ and α reported in the final column of Table 2 do not support the EACD ($\alpha = \kappa = 1$) or WACD ($\kappa = 1$) specification in preference to the GGACD.

The parameter values themselves support a relatively low autocorrelation

Table 4.2: Parameter Estimates for ACD(1,1) Models
of the NASDAQ and S&P 500 with Different Distributional Assumptions

Parameter	EACD(1,1)	WACD(1,1)	GGACD(1,1)
		NASDAQ	
ω	0.0196	0.0307	0.0570
	(0.0002)	(0.0018)	(0.0033)
γ_1	0.1158	0.1561	0.2121
	(0.0006)	(0.0041)	(0.0064)
ω_1	0.8720	0.8256	0.7779
	(0.0006)	(0.0051)	(0.0071)
α	-	0.5466	0.2714
		(0.0008)	(0.0052)
κ	-	-	3.4202
			(0.1165)
Ljung-Box Q(10)	239.0057	254.7124	258.3202
Ljung-Box Q(20)	372.6006	294.7861	280.1999
AIC	1.7020	0.8851	0.8694
SBC	1.7022	0.8854	0.8697
		S&P 500	
ω	0.0125	0.0182	-
	(0.0001)	(0.0003)	
γ_1	0.0834	0.0938	-
	(0.0002)	(0.0007)	
ω_1	0.9074	0.8881	-
	(0.0003)	(0.0007)	
α	-	0.6668	-
	-	(0.0007)	
κ	-	-	-
Ljung-Box Q(10)	1400.8710	1123.8400	-
Ljung-Box Q(20)	1971.5400	1489.7580	-
AIC	1.7293	1.3711	-
SBC	1.7294	1.3712	-

standard errors in (), all parameters are significant at the 1% level.

component to the conditional duration equation, with γ_1 less than 0.25. The clustering component, given by the parameter ω_1 is stronger at around 0.8 in each estimation. The general form of low autocorrelation and high clustering parameter estimates are common to existing literature estimating ACD models for IBM equities in Engle and Russell (1998), Disney stocks in Hautsch (2006) and US Treasuries in Dungey, Henry and McKenzie (2009). The shape parameter κ, from the GGACD(1,1) estimation supports a mixture of more than 1 Weibull distribution, while the α parameter suggests a smaller influence from the Exponential distributions. Thus far the results for the NASDAQ data support a GGACD(1,1) specification on the basis of the non-unit values of α and κ, although measures of fit suggest that a less complex distributional assumption provides a slightly better fit to the data.

The S&P 500 data has a far greater intensity than the NASDAQ data as described in Section 4.3, and the Ljung-Box coefficients are an order of magnitude higher than those reported for the NASDAQ. The estimated value of α in the WACD specification rejects the null hypothesis of $\alpha = 1$, which would support an EACD specification. In this case the GGACD(1,1) model failed to converge, producing extremely high estimates of κ, suggesting that there are problems remaining with the specification. The next section explores generalizations of these baseline specifications to examine the most likely means of improving the estimates.

4.5.2 Higher Order Lag Effects

Although many applications do find that ACD(1,1) models with varying distributional assumptions provide the best characterizations of their data, a small number of papers have favoured higher order lag lengths, (Dungey et al., 2009; Engle and Russell, 2005; Zhang et al., 2001). To explore lag effects, the WACD and GGACD specifications for the NASDAQ and the WACD specification for the S&P 500 are considered with extended lag lengths. Higher lag order ACD models are modelled in the order of increasing number of lag numbers, this process continues until the models fail to converge. The models are chosen based on Ljung-Box and AIC statistics of each model estimates. A similar process is not applied to the EACD models as none of the more general specifications reported in Section 4.5.1 support an Exponential distributional assumption.

The best results for the NASDAQ are a WACD(5,5) and GGACD(3,3) and are reported in Table 4.3. It is evident that the WACD(5,5) has reduced the Ljung-Box statistics considerably over the results reported in Table 4.2, and the sum of the estimated coefficients, $\sum_{j=1}^{5}\left(\gamma_j+\omega_j\right)\approx 0.9997$, indicates persistence in the adjusted durations. The unconditional mean adjusted duration for this specification is given by $E(\psi_i)=\omega/\left(1-\sum_{j=1}^{5}\left(\gamma_j+\omega_j\right)\right)\approx 3.0814$ seconds. It is notable that there is a drop in the value of the estimate of ω by two orders of magnitude compared with the WACD(1,1) specification from Table 4.2, but the shape parameter, α is unchanged to two decimal places.

The GGACD(3,3) specification contains some problematic outcomes. The Ljung-Box statistics are not reduced over the GGACD(1,1) specification, and importantly the sum of the ω_j and λ_j parameters, $\sum_{j=1}^{3}\left(\gamma_j+\omega_j\right)\approx 1.0000$, and the specific case where these parameters sum to unity is not encompassed in the GGACD model. The shape parameter values for α and κ are not greatly changed from the GGACD(1,1) specification. Of the two longer lag lengths investigated for the NASDAQ model the WACD(5,5) seems the more satisfactory.

Table 4.3: Parameter Estimates for WACD(5,5) and GGACD(3,3) Models of the NASDAQ

Parameter	WACD(5,5)	GGACD(3,3)
ω	0.0009	0.0030
	(0.0001)	(0.0003)
γ_1	0.2321	0.2977
	(0.0009)	(0.0063)
γ_2	-0.2510	-0.3261
	(0.0003)	(0.0081)
γ_3	0.0188	0.0465
	(0.0003)	(0.0022)
γ_4	0.0196	-
	(0.0007)	
γ_5	-0.0100	-
	(0.0006)	
ω_1	1.5840	1.5498
	(0.0001)	(0.0035)
ω_2	-0.5463	-0.5238
	(0.0002)	(0.0043)
ω_3	-0.0262	-0.0435
	(0.0002)	(0.0009)
ω_4	-0.0125	-
	(0.0001)	
ω_5	-0.0087	-
	(0.0004)	
α	0.5485	0.2784
	(0.0009)	(0.0005)
κ	-	3.2759
		(0.0038)
Ljung-Box Q(10)	183.7966	293.5161
Ljung-Box Q(20)	191.1481	306.5236
AIC	0.8799	0.7370
SBC	0.8807	0.7386

standard errors in (), all parameters are significant at the 1% level.

Specifications incrementing the lag lengths in the S&P 500 WACD(1,1) model fail to converge providing further evidence of the difficulties in fitting the S&P 500 data.

4.5.3 Volume Effects

As lag length adjustments have not made a substantial improvement to the model specifications, this section turns to the possible inclusion of other marks in the process; specifically, whether volume transacted has any extra information over the simple duration information. Bauwens and Veredas (2004) documented evidence of a significant relationship, but were restricted to daily volume proxies in their analysis. A further stream of literature, such as Bauwens et al. (2004) and Fernandes and Grammig (2006), considers the price durations, but given the difficulties with the unsigned price data in this sample, which introduces problems of bid-ask bounce requiring an approximating algorithm and associated uncertainty, this is left for future work.

Figures 4.2 and 4.3 suggest a negative relationship between volume and trade duration, an increase in volume transacted is associated with a decrease in trade duration, consistent with trade volume possessing information in this market, and that lack of trade indicates a lack of new information. The conditional duration equation (4.2) is augmented with the transacted volume information using the WACD(1,1) models reported in Table 4.2 as the baseline models.

Table 4.4 reports the results for the WACD(1,1) models for the NASDAQ and S&P 500 datasets augmented with volume information. In each case the volume parameter is negative and statistically significant at the 1% level. This result is consistent with the hypothesis that higher transacted volumes indicates some

Table 4.4: Parameter Estimates for WACD(1,1) Models with Volume for the NASDAQ and S&P 500

Parameter	NASDAQ	S&P 500
ω	0.0359	0.0227
	(0.0014)	(0.0031)
γ_1	0.1541	0.0938
	(0.0045)	(0.0068)
ω_1	0.8255	0.8857
	(0.0049)	(0.0072)
α	0.5470	0.6673
	(0.0007)	(0.0007)
v	-0.0011	-0.0007
	(0.0000)	(0.0000)
Ljung-Box Q(10)	246.6640	1093.8550
Ljung-Box Q(20)	284.1930	1422.3710
AIC	0.8841	1.3701
SBC	0.8844	1.3702

standard errors are given in parentheses
all parameters are significant at the 1% level.

form of information entering the market and shortening trade durations. There are two possible mechanisms for this outcome. In the first case public information may be causing market participants to reassess their positions and increasing trade intensity. In the second case, market participants observe increased trade volume and interpret it as an indicator of private information, and are hence encouraged to trade themselves, thus increasing trade intensities. Comparing the results with those reported in Table 4.2 there are few changes in the other parameter estimates. In particular, the shape parameter α, is little changed in either case. However, the Ljung-Box statistics have been improved by the inclusion of the additional volume mark.

4.5.4 Threshold Effects

While the NASDAQ data has been modelled in a way which may be considered acceptable, there remain considerable problems with the S&P 500 data. Since NASDAQ is less frequently traded, only S&P500 data sample is used to test threshold effects in this chapter. NASDAQ data could also be tested for threshold effects, however, the purpose of threshold effects modelling in this chapter is to provide a preliminary nonlinear ACD attempt. As shown in Section 4.3 there are some indications of different tail behaviours for large durations. Accounting for the possibility that these larger durations behave significantly differently to the bulk of the durations through a threshold model can significantly improve the model estimates. Zhang, Russell and Tsay (2001) found considerable improvements in estimates for the 3 months worth of IBM data examined in Engle and Russell (1998) by introducing non-linearities.

Table 4.5 reports the parameter estimates for a two regime threshold model with Weibull distribution TWACD(4,1:4,1) including the volume mark process as a further explanatory variable. That is, the complete model estimated is:

$$\psi_i = \begin{cases} \omega^{(1)} + \sum_{j=1}^{4} \gamma_j^{(1)} x_{i-1} + \omega_1^{(1)} \psi_{i-k} + v_i^{(1)}, \text{ if } 0 < x_i \leq r_1 \\ \omega^{(2)} + \sum_{j=1}^{4} \gamma_j^{(2)} x_{i-j} + \omega_1^{(2)} \psi_{i-k} + v_i^{(2)}, \text{ if } r_1 < x_i < \infty \end{cases} \tag{4.5}$$

where the regime cutoff, r_1 is chosen to be 19 seconds. A range of different alternatives from 10 to 40 was examined on the basis of the quantile-quantile[3] plot of the adjusted duration series in Figure 4.4. It is difficult to meet convergence for most of the attempts and cut off value 19 produced the most satisfactory

[3]The QQ plot applied here only provides a rough range of possible threshold values, we understand not much literature make use this plot on this problem. A grid search could provide a more accurate value but is rather time consuming.

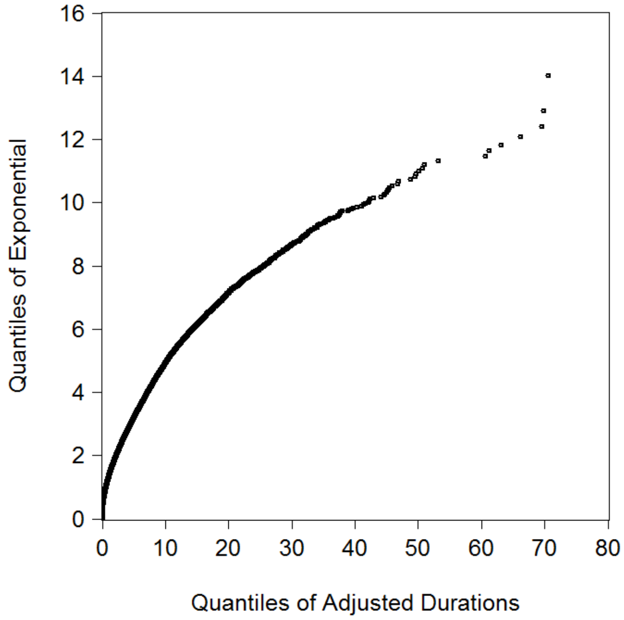

Figure 4.4: Quantile-Quantile plot of S&P500 Data

outcome. Note that this is a relatively large duration compared with the average adjusted duration of 1 second. Some 622 standardized duration observations exceed the cutoff point, leaving 683388 observations in the first regime. As the aim of threshold modelling in this chapter is to provide a preliminary attempt on non-linear ACD modelling, and the data sample is very large, the more complicated grid search is not carried on. A grid search could provide a more accurate value but is rather time consuming in terms of computational work and convergence problems.

The results in Table 4.5 show a remarkable improvement in the performance of the model compared with the WACD(1,1) for the S&P 500, with the Ljung-Box statistics dropping by a factor of 5, to levels commensurate with the models estimated for the NASDAQ data in earlier sections. The model supports the two thresholds, with quite distinct characteristics. In the first regime the mean adjusted duration is relatively small at 0.0312. The coefficient $\gamma_4^{(1)}$ is in-

Table 4.5: Parameter Estimates for ThresholdWACD(4,1:4,1) Model for the S&P 500

Parameter	estimate	standard error
$\omega^{(1)}$	0.0312	(0.0003)
$\gamma_1^{(1)}$	0.1653	(0.0018)
$\gamma_2^{(1)}$	-0.0486	(0.0022)
$\gamma_3^{(1)}$	-0.0173	(0.0019)
$\gamma_4^{(1)}$	-0.0016	(0.0013)
$\omega_1^{(1)}$	0.8744	(0.0007)
$\alpha^{(1)}$	0.6670	(0.0007)
$v^{(1)}$	-0.0009	(0.0000)
$\omega^{(2)}$	0.3718	(0.2322)
$\gamma_1^{(2)}$	0.0838	(0.0094)
$\gamma_2^{(2)}$	-0.1279	(0.0175)
$\gamma_3^{(2)}$	-0.0473	(0.0106)
$\gamma_4^{(2)}$	-0.0572	(0.0131)
$\omega_1^{(2)}$	1.3734	(0.0762)
$\alpha^{(2)}$	0.5827	(0.0158)
$v^{(2)}$	-0.0038	(0.0203)
Ljung-Box Q(10)	225.5850	
Ljung-Box Q(20)	261.6080	
AIC	1.3678	
SBC	1.3681	

standard errors are given in parentheses.

significant at 10% so that dropping that coefficient makes the preferred form a TWACD(3,1:4,1). The sum of the γ_j and ω_j coefficients in this first regime is 0.95, indicating considerable persistence. The volume coefficient, $v^{(1)}$ is negative and significant, indicating as previously that increased volume results in decreased trade duration.

In the second regime, however, a number of important differences are evident. Firstly, the mean duration, $\omega^{(2)}$ is increased 10 fold over the first regime, although this estimate is statistically insignificant. This could be due to the fact that the second regime contains values from a much larger range from around 19 to 70, and its number of observations is much smaller compared to the first regime. The role of volume with these longer duration transactions is also negative but is increased by over 6 times that of the first regime. The sum of the $\gamma_j^{(2)}$ and $\omega_1^{(2)}$ coefficients is greater than 1, due mainly to the estimate of $\omega_1^{(2)}$ indicating an extremely high degree of persistence in these right tail duration observations, a feature of the data which is not well handled by the standard model specifications.

The threshold ACD model provides a much improved description of the S&P 500 data than previous simpler specifications. There is a clear need to account for non-linearities in this dataset and a future research agenda would be to explore the use of mixture models such as De Luca and Gallo (2004) and Hujer and Vuletić (2007) and stochastic volatility duration models such as Ghysels, Gourieroux and Jasiak (2004) which hold promise of more flexibly incorporating the possibility of different regimes in the data.

4.6 Conclusions

This chapter provides, to the best of the author's knowledge, the first attempt to model the time between trade durations of an electronic after-hours equity futures market. The contributions of the chapter are the application to the previously unexploited after-hours electronically traded data, the use of a much longer data sample than previously explored in models of trade duration, and the use of volume as an informative mark. The preferred modelling framework is found to include relatively long lag lengths and threshold effects.

The markets studied comprise data from the standard NASDAQ and S&P 500 equity futures contracts traded on the Chicago Mercantile Exchange using data from the GLOBEX electronic trading platform during periods when the open outcry market for this contract is closed. The empirical results show that the trade duration of the equity market future contracts for the NASDAQ are characterized by relatively low autocorrelation and strong clustering, regardless of the distributional assumptions employed. In the S&P 500 data, the majority of the distribution also exhibits low correlation and high clustering, but large duration observations require a separate specification characterized by higher autocorrelation and no real clustering. The results show that the addition of volume information to the ACD model captures a statistically significant negative relationship between the trade duration and volume, consistent with either of two possibilities. The first of these possibilities it that public news results in large volume and high trade intensity as market participants adjust portfolios, and the second is that in the absence of public information, market participants interpret large volume trades as indicative of private information which feeds back to encourage further trading activity.

In terms of removing serial correlations from ACD model residuals, the estimation results are unsatisfactory, especially for the more liquid S&P 500 data sample. This brings up the need for addressing the nonlinearity within the data. In the following chapter, the same S&P500 data are used to investigate the nonlinearity problem, under nonlinear logarithmic ACD models.

Chapter 5

Logarithmic ACD Modelling

5.1 Introduction

In recent duration model studies, nonlinear ACD models have been found to be preferred to linear specifications. Traditional linear time series models often yield poor outcomes, especially in high frequency data studies. Results from linear ACD specifications in previous chapter further support these findings. The IBM intraday data used for the benchmark linear ACD model study in Engle and Russell (1998) was examined by Zhang et al. (2001), who found that even this 3-month period of data experiences a significant level of nonlinearity. In addition, Zhang et al. (2001) and Ghysels et al. (2004) suggest nonlinear ACD models outperform linear models on high frequency duration data.

The objective of this chapter is to build logarithmic ACD (log-ACD) models to estimate high frequency duration data in the after-hour electronic futures market. The log-ACD model was first proposed by Bauwens and Giot (2000), and was initially designed for relaxing the positivity restrictions on the parameters of a linear ACD model. Subsequently Bauwens et al. (2008) suggest that log-

ACD models also capture a nonlinear relationship between the duration and its lags. Bauwens et al. (2008) also study the moments of log-ACD models and compare volume durations using linear ACD and log-ACD models. The same paper provides analytical formulae for log-ACD moments. Nonlinearity is found to be only a minor problem since their data set is relatively small. However, when the data sample becomes large, it is more likely the data exhibits nonlinearity and potentially creates a significant problem to its corresponding linear ACD model. In this thesis, with a 2-year long sample of intraday high frequency data, the nonlinearity problem is too risky to ignore.

Apart from log-ACD models, there are many alternatives in the literature of nonlinear duration modelling. Some studies relax the distributional forms of the model, for instance Hujer and Vuletic (2006) develop discrete mixture duration models to capture more specific characteristics of intraday duration data. This discrete mixture model can be further extended by introducing of a more flexible discrete mixture-valued latent regime variable, see Bauwens and Veredas (2004). De Luca and Gallo (2004) use a mixture of two Exponential distributions to fit intraday duration data, relaxing the restrictions on the distributions of the error terms. The later developments in latent factor ACD models are often applied on stochastic volatility durations such as Ghysels et al. (2004).

Zhang et al. (2001) introduce the threshold ACD model to overcome the nonlinearity problem. In addition they applied Andrews and Ploberger structural break test to divide their data into a number of sub-periods to reduce nonlinear effects. However, they continue to find nonlinear dependence in the linear Weibull ACD (2,2) model within the sub-periods. Hence their proposed 3-regime threshold ACD model is applied individually in each of the sub-periods and yields a better result in term of capturing nonlinearity.

Compared with threshold ACD models and mixture of distribution models, log-ACD models are far less costly to estimate. This chapter demonstrates that log-ACD models are able to capture the nonlinearity problem without heavy model estimation and complicated procedures. The data set used in this chapter is the same S&P 500 data set from chapter 4. The less traded NASDAQ data are not used in this chapter in order to focus on the development of the log-ACD models. The S&P 500 data examined here covers 2 years, and as such it is very likely the data experiences nonlinearity. Further contribution of this chapter is to examine the performance of log-ACD models over a relatively long period of intraday data, across two forms of log-ACD models.

The rest of the chapter is constructed as follows. Section 5.2 describes the log-ACD model and its error term distributional forms. Section 5.3 gives some background of the after-hours electronic market. Estimation results are shown in Section 5.4. Section 5.5 discusses the effect of the volume and Section 5.6 concludes.

5.2 Model Description

The starting points of both traditional ACD and log-ACD models are similar, they both assume the duration process is a function of the conditional expected duration and an error term. If we let x_i be the duration, the duration process for a log-ACD becomes:

$$x_i = e^{\psi_i}\varepsilon_i, \tag{5.1}$$

where $e^{\psi_i} = \Psi$, which is the new conditional expected duration, and ε_i is an error term with many possible distribution forms. Common assumptions are Exponential, Weibull and Generalized Gamma.

The most general form of the log-ACD specification is:

$$\Psi = \ln E(x_i \mid I_{i-1}) = \omega_0 + \sum_{j=0}^{p} \gamma_j g(\varepsilon_{i-j}) + \sum_{j=0}^{p} \omega_j \Psi_{i-j}, \tag{5.2}$$

where I_{i-1} is the information set given at time t_{i-1}, ω_0 is a constant, ω_j and γ_j are coefficients for the past durations and past conditional expected durations, and $g(\varepsilon_{i-j})$ denotes a function of the error terms. There are two possible choices for $g(\cdot)$, and therefore two forms of log-ACD models are available. If we assume $g(\varepsilon_{i-j}) = \ln x_{i-1}$, the log-ACD model becomes:

$$\Psi = \omega_0 + \sum_{j=0}^{p} \gamma_j \ln x_{i-j} + \sum_{j=0}^{p} \omega_j \Psi_{i-j}, \tag{5.3}$$

where equation (5.3) denotes the log-ACD form 1 (log-ACD$_1$) model, and $g(\cdot)$ is set to be the logarithm of the past durations. By taking the exponential power, the logarithm of past durations converts back to observed past durations, although the past conditional expected duration Ψ will still be affected.

If we assume $g(\varepsilon_{i-j}) = \varepsilon_{i-j}$, together with equation (5.1) the log-ACD specification becomes:

$$\Psi = \omega_0 + \sum_{j=0}^{p} \gamma_j \frac{x_{i-j}}{e^{\Psi_{i-j}}} + \sum_{j=0}^{p} \omega_j \Psi_{i-j}, \tag{5.4}$$

where the new $g(\cdot)$ will be able to capture some level of nonlinearity when combined with equation (5.1). This new $g(\cdot)$ in the equation giving rise to the log-ACD form 2 (log-ACD$_2$) model. The log-ACD$_2$ model is often the standard model used in logarithmic ACD model studies. In fact, quite often the estimated results can be very different for these two forms, where the extent of difference between the two models depends on the significance of nonlinearity in the data. In this chapter, log-ACD$_2$ models are primarily investigated. The log-ACD models can be further categorized into different log-ACD forms, base on the functional form of the distribution of the error terms used. For example if ε_{i-j} is assumed to follow Exponential distribution, the model becomes an Exponential log-ACD model. Bauwens et al. (2008) study the moments of Exponential, Weibull, Gamma, Burr and Generalized Gamma distributional forms of log-ACD models.

Three forms of log-ACD$_2$ models are studied in this chapter: Exponential log-ACD(EL-ACD), Weibull log-ACD(WL-ACD) and Generalized Gamma log-ACD(GGL-ACD) models[1]. The GGL-ACD model by definition embeds or subsumes the other two models. The log likelihood function for GGL-ACD is:

$$\ell(x \mid \theta, x_{i_o}) = \sum_{i=i_o+1}^{T} \ln(\frac{\alpha}{\Gamma(\kappa)}) + \kappa\alpha \ln(\frac{x_i}{\lambda}) - \kappa\alpha(\ln x_i + \psi_i) - \Big(\frac{x_i}{\lambda e^{\psi_i}}\Big)^{\alpha}, \quad (5.5)$$

where $\lambda = \Gamma(\kappa)/\Gamma(\kappa + 1/\alpha), \theta = (\omega, \gamma_1, ..., \gamma_m, \omega_1, ..., \omega_q, \alpha)$, $x = (x_{i_o+1}, ..., x_T)$, and κ and α are gamma process parameters. When $\kappa = 1, \lambda$ reduces to $1/\Gamma(1 + 1/\alpha)$, and equation 5.5 will then reduce to a Weibull log likelihood function:

$$\ell(x \mid \theta, x_{i_o}) = \sum_{i=i_o+1}^{T} \alpha \ln\left[x_i\Gamma(1 + \frac{1}{\alpha})\right] + \ln(\frac{\alpha}{x_i}) - \alpha\psi_i - \left(\frac{\Gamma(1 + 1/\alpha)x_i}{e^{\psi_i}}\right)^{\alpha}, \quad (5.6)$$

where α is the Weibull parameter. When α is set to be 1, λ in equation (5.5) becomes 1, and equation (5.6) will become an Exponential log likelihood function

[1]The log-ACD$_1$ model results are shown in Appendix table 5.7, but with unsatisfactory results, therefore we focus on the three distribution form of log-ACD$_2$ models.

as below:

$$\ell(x \mid \theta, x_{i_o}) = -\sum_{i=i_o+1}^{T} \psi_i - \frac{x_i}{e^{\psi_i}}. \tag{5.7}$$

One of the advantages of the log-ACD model is that it allows negative variable coefficients to exist, which is especially important when adding additional market variables into ACD models with potentially negative coefficients. Traditional ACD models have non-negativity problems when adding negatively signed variables. In equation (5.2), $\Psi = \ln E(x_i \mid I_{i-1})$. Theoretically, as long as $E(x_i \mid I_{i-1}) > 0$, the parameters in equation (5.2) can be any number from negative infinity to positive infinity. The limitations on log-ACD model parameters are almost nonexistent although the bound $\left|\sum \omega_j\right| < 1$ is imposed for the purpose of covariance stationary of Ψ. As negative coefficients are accepted, the log-ACD model has significant advantages in incorporating additional marks such as volume or bid-ask quote.

The trade-off is that it seems very difficult to derive any analytical expressions for the unconditional moments. When we take the expectation of equation (5.3) or (5.4) we arrive that

$$\mu = E(e^{\Psi}) = E[e^{\omega_0 + \sum_{j=0}^{p} \gamma_j g(\varepsilon_{i-j}) + \sum_{j=0}^{p} \omega_j \Psi_{i-j}}], \tag{5.8}$$

which cannot be directly computed. Although Bauwens et al. (2008) provide an analytical formulae for the log-ACD model unconditional moments, the procedure is rather complicated. They derive that under the conditions of: $E[\exp(m\gamma\omega^{j-1} g(\varepsilon_i))] < \infty$, $\mu_m < \infty$ for an arbitrary positive integer m and

$|\omega_j| < 1$, the unconditional moments for log-ACD model follows:

$$E(x_i^m) = \mu_m \exp(\frac{m\omega_0}{1-\omega}) \prod_{j=1}^{\infty} E[\exp(m\gamma\omega^{j-1} g(\varepsilon_i))], \tag{5.9}$$

where $\omega = \omega_1$, and $\gamma = \gamma_1$ for log-ACD (1,1) model. In order to compute the unconditional moments, one does requires the knowledge of $E(\varepsilon^p)$ for log-ACD$_1$ models, and $E(\exp(p\varepsilon))$ for log-ACD$_2$ models. Since the moments of $E(\varepsilon^p)$ are available for the Generalized Gamma (including Exponential and Weibull) and Burr distributions, it is possible for one to derive the unconditional moments for the log-ACD$_1$ model. However, it is very complicated to derive the unconditional moments of log-ACD$_2$ models. The moment generating function provides $E(\exp(p\varepsilon))$, which is only available analytically for the Gamma distribution. To obtain an approximation of the moments for other distributions such as the Weibull, the Burr and the Generalized Gamma, one can use the Taylor expansion:

$$E(\exp(p\varepsilon)) = \sum_{k=0}^{\infty} \frac{p^k}{k!} E(\varepsilon^k), \tag{5.10}$$

where the infinite series of integer moments $E(\varepsilon^k)$ must converge to a finite value for $E(\exp(p\varepsilon))$ to exist. For the Burr distribution its moment is determined by a ratio of two shape parameters, therefore $E(\exp(p\varepsilon))$ does not exist. For the Weibull and the Generalized Gamma distributions, the unconditional moments do exist but only if its shape parameter is larger than one. So far it is obvious that it is possible but very complicated to derive the unconditional moments for log-ACD model, especially for log-ACD$_2$ form. Instead, recent literature such as Bauwens, et al. (2008) have focused on the over-dispersion ratios, autocorrelation functions (ACF), and differences between other moments of ACD models to explain the model.

5.3 Brief Market and Data Description

The after-hours S&P 500 futures data used in this chapter were collected from the Chicago Mercantile Exchange (CME) through the GLOBEX trading system[2]. The futures contracts traded after-hours in the electronic GLOBEX platform are the same contracts traded in the CME open outcry markets (the open outcry market only operates between 8:30 CST and 15:15 CST). During open outcry market trading time, the after-hours trading on GLOBEX is closed. It opens at 15:30 CST after the closure of the pit, and runs until 8:15 CST the next morning. The process continues for the rest of the week, except that on Fridays there are no electronic trades following the closure of the pit in the afternoon. In addition the electronic market trades reduced hours on US public holidays. There is a short period of closure everyday between 16:30 CST and 17:00 CST for maintenance of the GLOBEX electronic exchange system.

The electronic trading platform and the open outcry pit cover almost 24 hours of a day without overlap, thus market participants are able to exchange the same product in their portfolios in these indices at almost any time in a trading day. Apart from the standard sized contracts examined here, some other non-standard sized contracts, in particular the E-mini[3] contracts, trade almost 24 hours on the GLOBEX platform. Since the development of the electronic market in late 1990s, electronic trading has become more and more popular. Currently three contract

[2] GLOBEX is an electronic trading platform which runs continuously and is not restricted by borders or time zones.

[3] E-minis were introduced for the S&P500 in 1997 and for the NASDAQ in 1999. They have 1/5th size of a standard contract.

forms (standard future, electronic futures, E-mini) coexist in this market.

Since the opening hours for the after-hour electronic contracts are different from the standard contracts, traders may react differently to news announcements. According to Dungey, Fakhrutdinova and Goodhart (2009), the highest average volume in the after-hour period occurs immediately prior to the opening of the open outcry pit, peaking around 7:30 CST, which corresponds to the release time for prescheduled macroeconomic news in the US at 8:30 EST. Price impact for the S&P 500 contracts is also found to be lowest in the high volume period in that opening period, and higher in general during the European and Asian trading hours. Based on the liquidity of the market, the gains from information released can differ between the electronic and floor markets.

The intraday tick by tick data are from the S&P 500 futures contracts from July 1, 2004 to the end of September 2006. The trade durations are computed from the original tick transaction data set. All zero durations are removed by considering unique times, consistent with Engle and Russell (1998). A trade is treated as a transfer of ownership from one or more sellers to one or more buyers at a point in time, so that volume associated with transactions occurring at same time are aggregated. All negative durations are deleted. The data were diurnally adjusted prior to analysis in order to remove the typical time of day effect, see Engle and Russell (1998). Engle and Russell (1998) assume the deterministic seasonality effects act multiplicatively as

$$x_i = \widetilde{x}_i s(t_{i-1}), \tag{5.11}$$

where $\widetilde{x}_i$ is the seasonally adjusted duration, and $s(t_{i-1})$ is the seasonality component. The diurnally adjusted durations which are fractions above or below normal are expressed by taking ratios as $\widetilde{x}_i = \frac{x_i}{s(t_{i-1})}$. This is estimated by regressing the raw durations on the time of the day using a piecewise linear spline specification based on trade duration data with 1 hour intervals. There are 17 knots representing hourly intervals within a trading day, skipping the floor market trading hours. This leaves 684,010 observations for the S&P 500 data.

5.4 Estimation Results

Both forms of log-ACD (log-ACD$_1$ and log-ACD$_2$) models are applied to S&P 500 data. However, log-ACD$_2$ forms are the primarily focus here since the log-ACD$_1$ models provided unsatisfactory results. For log-ACD$_1$ specification where $g(\cdot)$ in equation 5.3 is assumed to be $\ln x$, the logarithmic effects will be lost when the exponential factor takes power of the expected duration on the left hand side. In fact the log-ACD$_1$ results are similar to the linear ACD models examined in chapter 4, which also fail to consider the nonlinearity in the data (Bauwens et al., 2008). As the degree of nonlinearity is large, the difference between the two forms of log-ACD models become larger and the misspecifications in log-ACD$_1$ model is also larger. The results for the log-ACD$_1(1,1)$ models are shown in Table 5.7 in the Appendix to this chapter. Results in the following sections are based on log-ACD$_2$ models.

The results from the Generalized Gamma log-ACD$_2$ model are not available due to failure of convergence. When it comes to a more general form like this, there are often too many parameters to maximize concurrently. Consequently, we present the results for the Exponential and Weibull log-ACD$_2$ models only. The

Table 5.1: Estimation Results from EL-ACD(2,2) form 2 model for S&P 500

Variable	Coefficient	Standard Error	Significance
ω	-0.0070	0.0000	0.0000
γ_1	0.1001	0.0004	0.0000
ω_1	1.7935	0.0017	0.0000
γ_2	-0.0939	0.0004	0.0000
ω_2	-0.7947	0.0017	0.0000
Ljung-Box Q(10)	86.2730		0.0000
Q(20)	132.9840		0.0000
AIC	1.7157		
SBC	1.7158		

Table 5.2: Estimation Results from WL-ACD(2,2) form 2 model for S&P 500

Variable	Coefficient	Standard Error	Significance
ω	-0.0084	0.0002	0.0000
γ_1	0.0779	0.0001	0.0000
ω_1	1.7681	0.0005	0.0000
γ_2	-0.0720	0.0001	0.0000
ω_2	-0.7700	0.0050	0.0000
α	0.6692	0.0007	0.0000
Ljung-Box Q(10)	104.2400		0.0000
Q(20)	169.2980		0.0000
AIC	1.3662		
SBC	1.3663		

Exponential and Weibull log-ACD(2,2) models both yield satisfactory results. The Ljung-Box Q(20) statistic is 132 for EL-ACD(2,2) model and 169 for WL-ACD (2,2) model from Table 5.1 and 5.2 , suggesting that serial correlations in the residuals are significantly removed. Tsay's nonlinearity test F-statistic is 1623 and significant at the 1% level, strongly indicating that there is nonlinearity in the raw durations. Table 5.3 compares the same estimates using the linear Weibull ACD(2,2) model; the Ljung-Box Q(20) statistics are much lower for log-ACD$_2(2,2)$ model. This suggests that when the nonlinearity problem of the data can be better addressed, the efficiency of ACD models can be further improved.

From this paragraph onwards, we refer log-ACD as the log-ACD model form 2 for simplicity. The differences between the Exponential and the Weibull log-ACD model results are minor. The EL-ACD(2,2) model has slightly better Ljung-Box Q(20) statistics and the WL-ACD(2,2) model has a more convincing AIC statistics (1.36 compared to 1.71). It is hard to choose which model is better, since both Ljung-Box and AIC statistics are essential criteria for ACD models. It leads to a question that what is really an accurate measure of goodness of fit of ACD models. Looking at the standardized residuals of ACD model is only one of many other methods for testing the model. Purely relying on either of the Ljung-Box or AIC statistics can be miss-leading. In this chapter the combination of these two criteria is used for the model selections, a better model needs to produce lower Ljung-Box and AIC statistics. Following such selection criteria, there is no clear advantage in either model as the model which produces lower Ljung-Box statistics has a higher AIC statistic. At this stage there is no preference between the two models.

Table 5.3: Comparison of a WL-ACD(2,2) (form 2) and a Standard Linear WACD(2,2) model for S&P 500

Variable	WL-ACD(2,2) Coefficient	WACD(2,2) Coefficient
ω	-0.0084	0.0016
γ_1	0.0779	0.1573
ω_1	1.7681	1.5627
γ_2	-0.0720	-0.1429
ω_2	-0.7700	-0.5788
α	0.6692	0.6695
Ljung-Box Q(10)	104.2400	297.7900
Q(20)	169.2980	319.5850
AIC	1.3662	1.3711
SBC	1.3663	1.3712

The α coefficient in Weibull log-ACD (2,2) model is 0.6692, suggesting that the distribution shape parameter is against a standard Exponential distribution. Note that there are some negative coefficients in Table 5.1 and Table 5.2, especially the unusual negative equation constant ω_0. This is common in log-ACD models since the only restriction on parameter coefficients is $\left|\sum \omega_j\right| < 1$. The $\sum \omega_j$ for EL-ACD(2,2) and WL-ACD(2,2) are 0.9988 and 0.9981 respectively.

Table 5.4: Over-dispersion Statistics
Before and After Log-ACD(2,2) Estimates

	raw data	EL-ACD(2,2)	WL-ACD(2,2)
Observations	684010	684010	684010
Mean	0.9973	0.9999	1.0301
standard deviation	1.8976	1.6927	1.7456
over-dispersion ratio	1.9027	1.6928	1.6946

High frequency financial data often experience over-dispersion[4], where the standard deviation is larger than the mean. The S&P 500 data also experience the same problem in all models examined in this chapter. Table 5.4 lists the dispersion statistics before and after the Exponential and Weibull log-ACD(2,2) models. The over-dispersion ratio used in Bauwens and Giot (2000), has improved from 1.9 to around 1.6 after the log-ACD_2 estimates in this chapter. However it is still larger than 1, indicating that the data still experience over-dispersion. The over-dispersion ratio for open outcry markets data are usually around 1.3, see Bauwens et al. (2008). From the results in this chapter, it is likely that the after-hours electronic market data experiences a larger degree of over-dispersion than the pit market data.

The normalized series is shown in Figure 5.1, where panel (a) is the diurnally adjusted duration and panel (b) is the normalized innovations of our Weibull

[4]The over-despersion ratio is standard deviation divided by the mean.

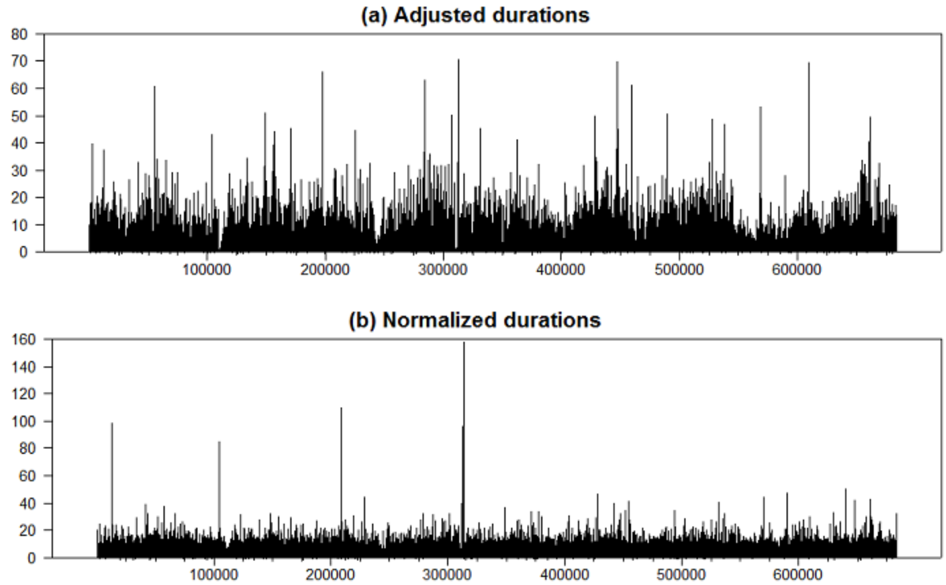

Figure 5.1: (a) Adjusted Durations and (b) Normalized Durations after WL-ACD(2,2) model.

log-ACD(2,2) model. Four major spikes with relatively large magnitude can be seen in panel (b). It is quite unusual to have such abnormal spikes after estimation, therefore we look back into these individual dates. It is found that most of the unusual durations are around 7:30 CST. It is well known that this time corresponds to macro news announcements in the US. However, the detailed causes for the spikes in these particular dates remain unknown. No major events nor news announcements corresponding to each particular day are found except for the last spike on July 7th, 2005. On that day, four explosions were reported on the London underground and bus system leading to some transportation networks being shut down (the 7th July London terrorist attack). In a similar manner to public holidays, these abnormal data points should be removed. The revised normalized series is shown in Figure 5.2.

The Weibull log-ACD (2,2) model has been re-estimated based on the revised data, with the results shown in Table 5.8 in Appendix of this chapter. The coef-

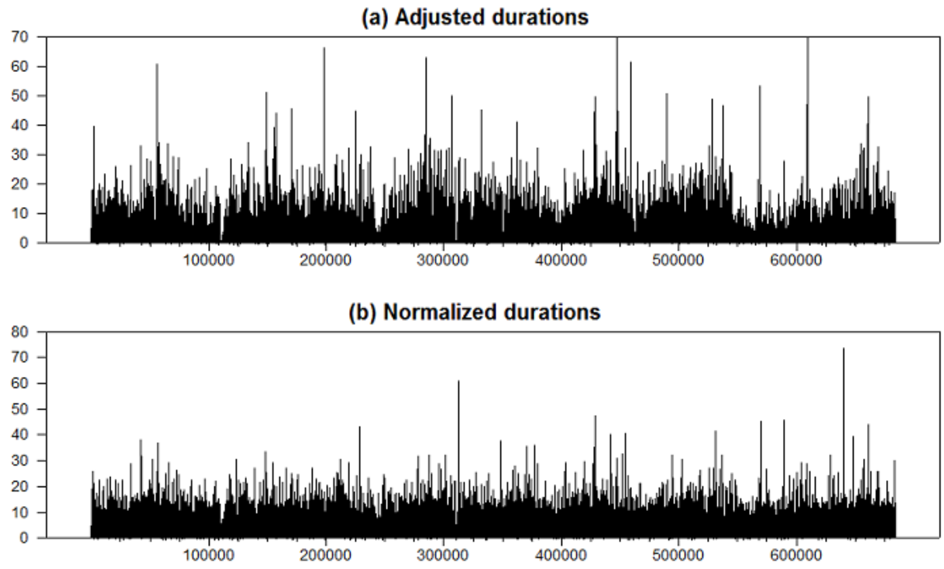

Figure 5.2: (a) Adjusted Durations and (b) Normalized Durations after WL-ACD (2,2) Model with Volume Based on Revised Data

ficients show only minor differences from those reported previously. The model is improved slightly, with a reduced Ljung-Box statistic. No major differences have been found. The model estimates in the next sections are all based on the revised data. The over-dispersion ratio has improved slightly from 1.69 to 1.66 for the revised data, as shown in Table 5.5

Table 5.5: Over-dispersion Statistics Before and After Weibull Log-ACD(2,2) Estimates based on revised data

	raw data	WL-ACD(2,2) with revised data
Observations	684010	684010
Mean	0.9973	1.0293
standard deviation	1.8976	1.7158
overdispersion ratio	1.9027	1.6670

5.5 Volume Effects

Since the Exponential log-ACD and Weibull log-ACD both provide similar results, only the Weibull logACD model is chosen for the study of volume effects.

Volume is added as an additional variable to the model, WL-ACD (2,2) model and the results are shown in Table 5.6. The coefficients for lag durations and lag conditional expected durations only slightly differ to the WL-ACD(2,2) model in the initial estimates reported in Table 5.2. The Ljung-Box Q(20) has major improvement from 169 to 97 and the AIC and SBC statistics also improved slightly. The volume coefficient is a small negative number, it suggests there is a negative effect from volume on durations. This is consistent with the market microstructure theory, where high volume corresponds with heavy trading activities. Easley and O'Hara (1992) argue that longer durations correspond with uninformed trading and imply the stock value has not changed, hence less trading took place within the particular time period. Therefore it is more likely to have low volume in the market when the duration is relatively long. On the other hand, when duration is short, the intensive trading activities imply that asymmetric information is available in the market and the value of the particular asset is changing quickly. In this short period of value adjustment of the asset, high volume occurs since investors prefer to take advantage of their information available until the asset price is fully adjusted. The results in this chapter support the same theory applying in the after-hours futures market. The over-dispersion ratio improves to 1.66 compared to the previous ratio 1.69 in last section. By adding volume to the model and using the revised data, the model specification has improved.

The autocorrelation functions for the adjusted duration and normalized innovations with 20 lags are presented in Figures 5.3 and 5.4. A very long tail found in the ACF graph in Figure 5.3 indicates the data display long memory. From

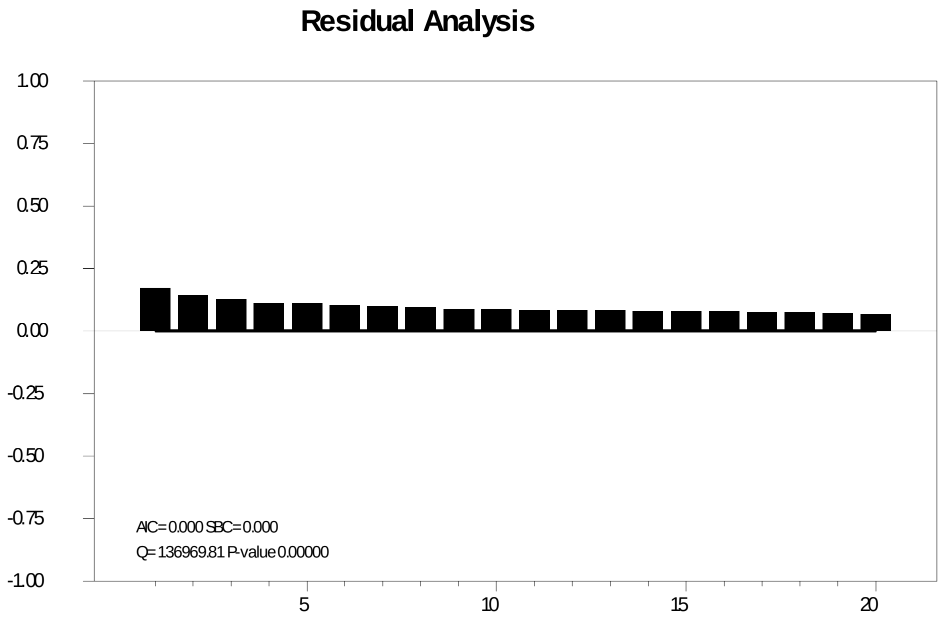

Figure 5.3: Autocorrelation Function for the Adjusted Duration Series with 20 lags

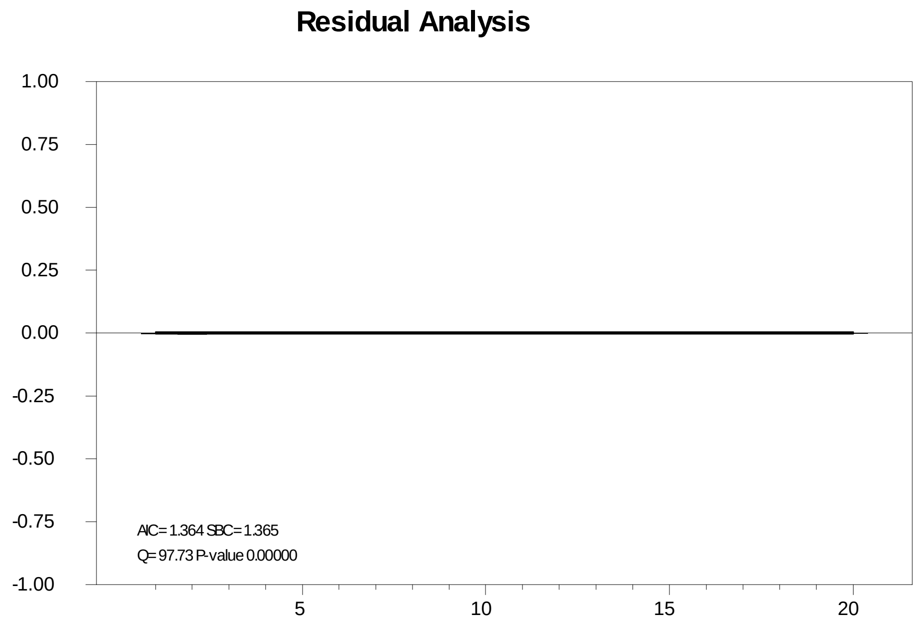

Figure 5.4: Autocorrelation Function for the Normalized Innovation Series with 20 lags

Table 5.6: Estimation Results from WL-ACD(2,2) for Revised S&P 500 Data with Volume

Variable	Coefficient	Standard Error	Significance
ω	-0.0124	0.0004	0.0000
γ_1	0.0813	0.0007	0.0000
ω_1	1.6912	0.0071	0.0000
γ_2	-0.0718	0.0007	0.0000
ω_2	-0.6949	0.0070	0.0000
α	0.6700	0.0007	0.0000
ν	-0.0004	0.0000	0.0000
Ljung-Box Q(10)	81.7470		0.0000
Q(20)	97.7400		0.0000
AIC	1.3645		
SBC	1.3646		

Figure 5.3, it is clear that the immediate correlations are obviously different to later lags, but after the 2nd or 3rd lags, the correlation effects become very similar. Historical influence from the 10th and 15th lags are almost the same. Figure 5.4 shows the ACF for the normalized innovation series. Clearly, the normalized innovations have no significant serial correlations. The long memory pattern is commonly found in high frequency intraday data, especially in duration data, as described in earlier chapters. However, long memory could also arise by ignoring structural breaks within the sample. Further analysis on potential structural breaks problem is presented in later chapters (chapter 6 and 7).

5.6 Conclusions

In this chapter, log-ACD models are used to study the after-hours electronic futures market. Two forms of log-ACD models are studied and log-ACD$_2$ is found to fit better for S&P 500 data. Log-ACD$_1$ models perform poorly, especially when the degree of nonlinearity is large. Both Exponential and Weibull log-ACD form

2 models yield satisfactory results. However, there are still some event effects or abnormal data points apparent, which are subsequently identified and removed in later estimation.

Volume was added as an additional factor in the model and produced an improved model both in terms of removing serial correlations and fitting the data. The after-hours electronic futures data used in this chapter failed to pass the F-test of linearity indicating very strong nonlinearity, the linear Weibull ACD models yield poor results compared with the Weibull log-ACD$_2$ model. Although a more precise log-ACD$_2$ model (with higher lag order or more general distributional assumptions) is still required to fully capture the nonlinearity problem, it is shown that the sample log-ACD$_2$ model is able to handle a certain degree of nonlinearity. Compared with the traditional linear form of ACD models, the log-ACD$_2$ models have much improved estimates from the after-hours electronic futures market data. Finally, the Weibull log-ACD$_2$ (2,2) model is found to be the best log-ACD$_2$ models considered for the data in this chapter. The log-ACD model residuals continue to find over-dispersion. Unfortunately, it is very difficult at this juncture to interpret the unconditional moments of the Weibull log-ACD$_2$ model.

In the following chapter, we build another formulation of nonlinear ACD models to better model duration patterns in the after-hours electronic futures market by including structural breaks.

5.7 Appendix

Table 5.7: Log-ACD form 1 Results for Exp log-ACD(2,2)

Variable	Coefficient	Standard Error	Significance
ω	0.0051	0.0001	0.0000
γ_1	0.1156	0.0008	0.0000
ω_1	1.6704	0.0035	0.0000
γ_2	-0.0939	0.0007	0.0000
ω_2	-0.7947	0.0034	0.0000
α	0.6601	0.0007	0.0000
Ljung-Box Q(10)	5394.2360		0.0000
Q(20)	9484.736		0.0000
AIC	1.3772		
SBC	1.3773		

Table 5.8: Estimation Results After Removal of the Spikes for S&P 500 (WL-ACD (2,2) form 2)

Variable	Coefficient	Standard Error	Significance
ω	-0.0093	0.0003	0.0000
γ_1	0.0789	0.0007	0.0000
ω_1	1.7585	0.0053	0.0000
γ_2	-0.0724	0.0006	0.0000
ω_2	-0.7606	0.0053	0.0000
α	0.6697	0.0007	0.0000
Ljung-Box Q(10)	110.4090		0.0000
Q(20)	156.0910		0.0000
AIC	1.3649		
SBC	1.3650		

Chapter 6

ACD Modelling with Structural Breaks (Pre-Crisis Period)

6.1 Introduction

The traditional linear ACD from Engle and Russell (1998) has been suggested to perform poorly where data experiences nonlinearity or structural breaks (Zhang et al., 2001; Ghysels et al., 2004). Through the development of ACD models, the non-linearity problem commonly observed in duration data has been extensively addressed. In the previous chapter, we examined after-hours S&P 500 data sample using logarithmic ACD models from Giot (2000). As addressed by Zhang et al. (2001), Mikosch and Starica (2004), Hillebrand (2005), and many other papers, nonlinearity and long memory patterns could also occur through ignoring structural change effects. Hence this chapter studies the structural stabilities of the ACD model parameters for the same S&P 500 duration data examined in the previous chapter.

Structural breaks based on linear models using least squares have been well studied in the literature, but relatively few structural breaks studies are based on more general conditional models, particularly through maximum likelihood in the ACD framework. Many of these ACD models are based on the strong similarity to GARCH models, and long memory features in the estimated data are commonly found. The long memory pattern observed could be caused by ignoring potential structural breaks within the model. However, most studies on nonlinearity, structural breaks, or threshold effects using ACD models which are more flexible and less restrictive, only involve a relatively short sample period, for example the 3 month data sample in Zhang et al. (2001). This chapter studies a two-year period of intraday high frequency data between 2004 and 2006, making any potential structural break problem hard to ignore.

The after-hours electronic markets only became popular in early 2000s. Among existing literature, this chapter provides an unique structural break study based on the after-hours electronic futures market duration data. It is particularly interesting not only to see how the structural breaks affect ACD models, but also how the structures behave in the after-hours futures market in the period examined.

In this chapter, the LM-based structural break test of Andrews (1993) and Andrews and Ploberger (1994) are applied to a Weibull ACD (WACD) model to test for structural breaks. The sample data is then segmented into sub-periods and studied individually using a WACD. An alternative approach would be to use regime switching models but we do not pursue this as it requires us to exogenously specify the number of regimes. By considering structural breaks in the sample data, the nonlinearity problems outlined in previous chapters for estimating ACD model are being addressed. The chapter is constructed as follows: Section 6.2

gives a brief literature review on structural breaks detection; Section 6.3 and 6.4 present data description and methodology; Section 6.5 illustrates the results and finally Section 6.6 concludes.

6.2 Brief Review of Structural Break Tests

Ignoring structural breaks can be costly. Mikosch and Starica (2004) provide evidence of false long memory in the autocorrelation function if one ignores structural breaks. Andreou and Ghysels (2009) also summarize the problems of ignoring structural breaks in financial time series, such as false integrated models that yield long memory in the autocorrelation function.

Hillebrand (2005) provides a theoretical explanation for this effect, where by ignoring the structural changes, a spurious integrated GARCH process is obtained. By comparing the source of stochasticity in GARCH and AR processes, Hillebrand (2005) concludes that GARCH models are more sensitive to change points than AR processes. Since ACD models and GARCH models share many similar properties, ACD models should also be expected to be sensitive to structural breaks. Diebold (1986) and Lamoureux and Lastrapes (1990) also highlight the consequences of unaccounted structural breaks and regime switches in financial time series.

There are many ways to categorize the existing change-point tests in the financial time series literature, here we only introduce literature relevant to tests based on multiple unknown change points in ACD models. The traditional optimal tests such as in Farley and Hinich (1970), and Kim and Siegmund (1989) do not apply in the case when the change points are unknown, because the nuisance parameter is not identified under the null hypothesis and the test statistics are not in a standard distributional form. This problem was not solved until Andrews (1993) developed the sup-LM test and tabulated critical values of the test statistics. Further developments includes Andrews and Ploberger (1994, 1996), Hansen (1996), Sowell (1996), Bai and Perron (1998), and Hall and Sen (1999). The test from Bai and Perro (1998) allows multiple structural breaks to be tested at the same time, but the exact number of breaks within the period is required to be known. Smith (2008) conducts Monte Carlo experiments and finds that the traditional diagnostic tests such as Wooldridge (1990) robust LM tests for autocorrelation failed to detect structural breaks in GARCH related models.

Two of the more popular structural break tests in recent literature are the CUSUM break test of Inclan and Tiao (1992), and the LM-based structural break tests of Andrews (1993) and Andrews and Ploberger (1994).

The CUSUM test of Inclan and Tiao (1992) was originally designed for the problem of multiple change points caused by a change in the variance of a sequence of independent processes. The algorithm for detecting the variance changes is to use an iterated cumulative sums of squares. CUSUM-type tests have been extended in the recent literature and applied to strong mixing processes (Kokoszka and Leipus, 2000). Smith (2008) points out that when returns experience fat tails, the CUSUM test tends to over-reject structural breaks.

The LM-based tests on the other hand, developed by Andrews (1993) and Andrews and Ploberger (1994), have more accurate size and better power to detect a range of breaks in dynamics of conditional volatility. The LM based tests used to detect breaks in this chapter are designed for testing unknown breaks. The SupLM test is inspired by Davies (1987) and has the limitation that it depends on only one sample point. Andrews and Ploberger (1994) developed the Exponential Lagrange Multiplier statistic (ExpLM) and Weighted Averages of LM tests (AveLM) to improve the power of the LM-based structural break tests. Their tests have since been extended from linear regression models to more general models. For example, Sowell (1996), and Hall and Sen (1999) extend tests for parameter instability to the GMM framework, Hansen (1998) tests structural changes in conditional models.

One problem of the LM-based tests is that if the break point is too close to the boundary, say the beginning 15% or final 15% of the data, the LM-based tests can yield poor results (Andrews, 1993). Additionally, there are a number of cautions when applying these break tests. Vogelsang (1997) find that applications with CUSUM and LM-based tests may overestimate the number of breaks and find incorrect locations.

There are also many other forms of structural break tests, namely the generalized fluctuation tests in Kuan and Hornik (1995) and Leisch et al. (2000). Davis et al. (2005) present a method for detecting the optimal number and location of multiple change points in stochastic volatility process based on a minimum description length criterion.

6.3 Data Description

The data were collected from Chicago Mercantile Exchange (CME) and comprise trades posted on the GLOBEX trading system[1] as intraday tick by tick on the

[1]GLOBEX is an electronic trading platform which runs continuously and is not restricted by borders or time zones.

S&P 500 futures contracts from July 1, 2004 to the end of September 2006. The trade duration data are computed from the original tick transaction data set. Consistent with Engle and Russell (1998), all zero durations are removed by considering unique times. A trade is recognized as a transfer of ownership from one or more sellers to one or more buyers at a point of time, the volume associated with transactions occurring at the same time are aggregated. The data were diurnally adjusted prior to analysis in order to remove the typical time of day effect, see Engle and Russell (1998).

Engle and Russell (1998) assume the deterministic seasonality effects act multiplicatively. Similarly to previous chapters, the sample data is diurnally adjusted using spline based on 1 hour intervals. Skipping the floor market trading hours, there are 17 knots representing hourly intervals within a trading day. The diurnal adjusted duration series includes 684,004 observations and experiences long memory in its ACF. The ACF is decays at a very slow rate as shown in Figure 6.3 in the Appendix to this chapter.

6.4 Methodology

The test used for detecting structural breaks in this chapter is the LM-based tests of Andrews (1993) and Andrews and Ploberger (1994), henceforth AP test. Similar to the CUSUM test, this LM-based methodology falls into the category of binary and sequential sample segmentation class of structural break test. Basically it treats the multiple change points detection as an extension of the single change point problem. It first tests the total sample of the data, then if a change point is detected, the data sample is segmented into two sub-samples and retested.

This process is continued until no further change points are detected. By applying these LM-based tests, it is possible to estimate the number and location of breaks.

Consider a time series data y_t for $t = 1, ...T$ parameterized by θ_t. Use $\pi \in (0, 1)$ as the percentage location of the sample data within the change point. The process becomes two, with parameters θ_1 and θ_2 when there is a break:

$$\theta_t = \left\{ \begin{matrix} \theta_1(\pi) & \text{for } t = 1, ...[T\pi] \\ \theta_2(\pi) & \text{for } t = [T\pi], ...T \end{matrix} \right\}, \tag{6.1}$$

where $[T\pi]$ is the proportion of sample occurring before the break, rounded to the nearest integer.

The null hypothesis of no structural breaks is that the parameters in all periods are the same:

$$H_0 : \theta_t = \theta_0 \ . \tag{6.2}$$

In the case when the break point is already known, the problem is very simple, breaks can be detected by using a standard Chow test. Alternatively one can use a standard LM test for structural break:

$$LM_T(\pi) = \frac{T}{\pi(1-\pi)} \overline{g}_{1T}(\widetilde{\theta}, \pi)^T * S_T^{-1} D_T (D_T^T S_T^{-1} D_T)^{-1} D_T^T S_T^{-1} \overline{g}_{1T}(\widetilde{\theta}, \pi), \tag{6.3}$$

where $g(y_t; \widetilde{\theta}) = \partial \log f(y_t; \widetilde{\theta}) / \partial \widetilde{\theta}$, which is the partial derivative of the log density of parameter vector; $\overline{g}_{1T} = (1/T) \sum_{t=1}^{T} g(y_t; \widetilde{\theta})$, with

$$S_T = (1/T) \sum_{t=1}^{T\pi} (g(y_t; \widetilde{\theta}) - \overline{g}_{1T}(\widetilde{\theta}))(g(y_t; \widetilde{\theta}) - \overline{g}_{1T}(\widetilde{\theta}))^T, \tag{6.4}$$

and $D_T = (1/T)\sum_{t=1}^{T}(\partial g(y_t;\widetilde{\theta})/\partial\widetilde{\theta}^T)$.

However, most of the time the change point is unknown. Standard distributional theory can not be applied here. The null hypothesis of no structural break becomes singular information matrix, and the log-likelihood will be the same for all possible break points when $\theta_1 = \theta_2$. In such a context, Andrews (1993) introduce the (Average) Exponential LM test and reported the critical values of the non-standard distribution. The optimal test statistic *Exp-LM*$_T$ is defined by

$$Exp\text{-}LM_T = (1+c)^{-p/2}\int \exp(\frac{1}{2}\frac{c}{1+c}LM_T(\pi))dJ(\pi), \tag{6.5}$$

where p is the dimension of parameter set θ. A weight function $J(\cdot)$ is applied here over values of π in Π (Π is the space of the full data sample, ie $\Pi = [0,1]$), and c>0 is a scalar constant that depends on the $J(\cdot)$ function. The value of the constant c determines whether the power of the alternative is far or close within the sample.[2] Andrews and Ploberger (1994) suggest that among the class of all tests of asymptotic significance level α, the *Exp-LM*$_T$ test has the best weighted average power asymptotically. At asymptotic level α, the *Exp-LM*$_T$ test maximizes

$$\lim_{T\to\infty}\int P(\varphi_T \text{rejects}|\theta_0 + B_T^{-1}h, \pi)dQ_\pi(h)dJ(\pi) \tag{6.6}$$

over all tests φ_T, where B_T is a nonrandom $s\times s$ diagonal matrix and s is the dimension of the new parameter of the next regime, so that $s = p + q$. A weight function $Q(\cdot)$ for h is introduced here to measure the extra weights in the

[2] The Exp-LM$_T$ test depends on c unless in the special case of $J(\pi)$ is a pointmass distribution.

parameter changes, where h follows $\theta_2 = \theta_1 + h$. In equation (6.5), there are two extreme cases according to values of the scalar constant c. If $c \to 0$, the Exponential LM test reduces to the "Average LM" test statistic (*Ave-LM*). The *Ave-LM* follows

$$\lim_{c\to 0} 2(Exp\text{-}LM_{TC} - 1)/c = \int LM_T(\pi)dJ(\pi). \tag{6.7}$$

The *Ave-LM* statistic is especially useful for testing alternatives which are very close to the null hypothesis. This is because when $c \to 0$, less weight is given to the alternatives for a large structural break. However, when limit is set towards the other extreme as $c \to \infty$, the Exponential LM statistics take on an Average Exponential form:

$$\lim_{c\to\infty} \log((1+c)^{p/2} Exp\text{-}LM_{TC}) = \log \int \exp(\frac{1}{2}LM_T(\pi))dJ(\pi), \tag{6.8}$$

which is particularly useful for testing against more distant alternatives. The *Sup-LM* statistic can be generated from Exp-LM statistic by replacing the constant term $\frac{c}{1+c}$ with another constant $r > 0$. When $r \to \infty$, the *Sup-LM* statistic is defined as:

$$\lim_{c\to\infty} (\log Exp\text{-}LM_T^r)/r = \sup_{\pi\in\Pi^*} LM_T(\pi), \tag{6.9}$$

where $\Pi^* \subset \Pi$ and Exp-LM_T^r is the Exp-LM_T statistic with $\frac{c}{1+c}$ replaced by r. Andrews (1993) find that the *Sup*-LM test performs poor results if $\Pi = [0, 1]$. By dropping a proportion of the sample data, estimate results can be improved. According to Andrews (1993), the boundary was defined at 15%. *ie* $\Pi = [\pi_0, 1 - \pi_0]$, and $\pi_0 = 15\%$.

Since the *Sup-LM* test only depends on one sample point, the power of the test is limited. The distribution of the *Sup-LM* test depends on the proportion of sample dropped and on the number of parameters. The *Ave-LM* test and *Exp-LM* test, however, are possible to consider all the break points for the structural break test.

The model used in this chapter is the Weibull ACD (WACD) model of Engle and Russell (1998) and is described as follows. If trades in a particular market occurred at time t_i, the duration which is the irregular interval between consecutive trades $x_i = t_i - t_{i-1}$, where t_{i-1} is the immediately previous trade. The duration, x_t, is diurnally adjusted before entering the model. After removing the daily pattern the expectation of the ith duration is written by

$$\psi_i \equiv E(x_i \mid x_{i-1}, ..., x_1) = \psi_i(x_{i-1}, ..., x_1; \theta),$$

where $\psi_i \equiv E(x_i|x_{i-1,}, \ldots, x_0)$ represents the conditional expected duration and θ denotes the parameter space. Duration x_i is assumed to follow

$$x_i = \psi_i \varepsilon_i \ , \tag{6.10}$$

where $\{\varepsilon_i\} \backsim i.i.d.$[3] and ε_i is an error process. The clustering and autoregressive aspects of duration can then be captured by the specification of conditional expected duration. The basic ACD(p, q) model can be written as:

$$\psi_i = \omega + \sum_{j=0}^{p} \gamma_j x_{i-j} + \sum_{k=0}^{q} \omega_k \psi_{i-k}, \tag{6.11}$$

[3] $\{\varepsilon_i\}$with density $p(\varepsilon; \phi)$,$\phi$and θ are variation free

where ω, γ_j and ω_k are parameters, and p and q represent the lag orders, see Engle and Russell (1998). For a Weibull ACD, the error term ε_i is assumed to follow a Weibull distribution. The conditional log likelihood function for a WACD model is

$$\ell(x \mid \theta, x_{i_o}) = \sum_{i=i_o+1}^{T} \alpha \ln \left[\Gamma(1+\frac{1}{\alpha}) \right] + \ln(\frac{\alpha}{x_i}) + \alpha \ln(\frac{x_i}{\psi_i}) - \left(\frac{\Gamma(1+1/\alpha) x_i}{\psi_i} \right)^{\alpha}, \tag{6.12}$$

where $x_i \geq 0$ and $\alpha > 0$ is a shape parameter for a Weibull distribution. The WACD model is estimated by maximum likelihood of equation (6.12). Other models such as log-ACD and Generalized Gamma ACD models could also be considered as in previous chapters. However, the log-ACD model appears to produce very unstable estimates in the presence of substantial non-linearities and it is very difficult to obtain convergence for Generalized Gamma ACD models[4]. Consequently a Weibull ACD (1, 1) is used for structural breaks study in this chapter. Higher lag orders of Weibull ACD models are not pursued in this chapter for the following reasons: first, low lag orders ACD models are far less costly to estimate than higher order ACD models; and second, higher lag orders ACD models can produce some individual negative coefficients, as long as $\sum_{j=0}^{p} \gamma_j > 0$, $\sum_{k=0}^{q} \omega_k > 0$, $\sum_{j=0}^{p} \gamma_j + \sum_{k=0}^{q} \omega_k < 1$ are satisfied. These negative coefficients may cause problems in the residuals and conditional expected durations. More importantly, it is more likely for a model with some negative coefficients to reach a local maximum instead of the global. These parameter limits problems have been addressed in Bauwens and Giot (2000).

[4] As discussed similarly in chapter 4.

6.5 Structural Break Application Results

In this section, the Andrews Ploberger test is applied to the WACD model parameters. The S&P 500 sample data is the same as previous chapters from 1st July 2004 to 29th September 2006. Following chapter 4, the model specification is a Weibull ACD (1, 1) with volume as an additional mark[5]. The ACD model parameters are fitted into the AP break test in a consistent manner with the GARCH break test literature. A Weibull ACD(1,1) with volume is estimated as follows:

$$\psi_i = \omega + \gamma_1 x_{i-1} + \omega_1 \psi_{i-1} + v_1 vol_{i-1}, \tag{6.13}$$

where vol_{i-1} is the past volume parameter. The AP break test examines the structures of the conditional expected duration ψ_i, constant ω, lagged duration x_{i-1}, lagged conditional expected ψ_{i-1}, and volume parameter v_1. The *Sup*-LM and *Ave*-LM break test statistics for all coefficients are selected to determine the break point locations. After the break has been detected, Weibull ACD(1,1) models are fitted for the sub-periods, and the AP test is reapplied to detect additional breaks. As ultra high frequency data lasts 2 years, a 22 day[6] filter is applied for the detection process. All qualified test statistics have to be significant at 1% level. Note that it is still possible for a shorter sub-period (less than 22 days) to occur, as a break point is found to be located close to the end point of previously defined period.

Nineteen break points are detected using the above filters using twenty-seven months data, averaging just over a month for every subperiod. Table 6.1 shows

[5] Other marks such as bid-ask spread, price are not added due to lack of data.

[6] 22 days of S&P 500 intraday data have around 20,000 data points and is normally treated as a working month.

Table 6.1: Three Major Break Level Statistics
from A Weibull ACD(1,1) Model for The S&P 500 Data

		(01/07/04-11/04//05) Break Date: 29/09/04 aveLM=46.19 (0.00) sup LM=103.70 (0.00)
	(01/07/04-12/05/06) Break Date: 4/11/05 aveLM=112.16 (0.00) sup LM=239.03 (0.00)	
		(12/04/05-12/05/06) Break Date: 06/01/06 aveLM=32.48 (0.00) sup LM=76.99 (0.00)
(01/07/04-29/09/06) Break Date: 12/05/06 aveLM=185.34 (0.00) sup LM=385.09 (0.00)		
		(13/05/06-16/06/06) -No Break-
	(13/05/06-29/09/06) Break Date: 16/06/06 aveLM=174.07 (0.00) sup LM=362.12 (0.00)	
		(17/06/06-12/05/06) Break Date: 07/07/06 aveLM=13.59 (0.00) sup LM=35.78 (0.00)

Date follows day/month/year order

the detection process for the first four major break points. Figure 6.1 shows the hierarchical orders of the detection process, with level 1 representing the first break detected. The maximum of the LM, which is the *Sup-LM* test statistic, of 385.09 is obtained on 12th May 2006. Back in early May, 2006, Kirkland, Washington based Merit Financial Inc. filed for bankruptcy. Since then, the potential subprime crisis started build up though out late 2006 and early 2007. This significant break point located for the sample data further incorporated the changes in market structure from duration's perspective. Figure 6.2 is the plot of Andrews and Ploberger test statistics over the whole sample, and shows at

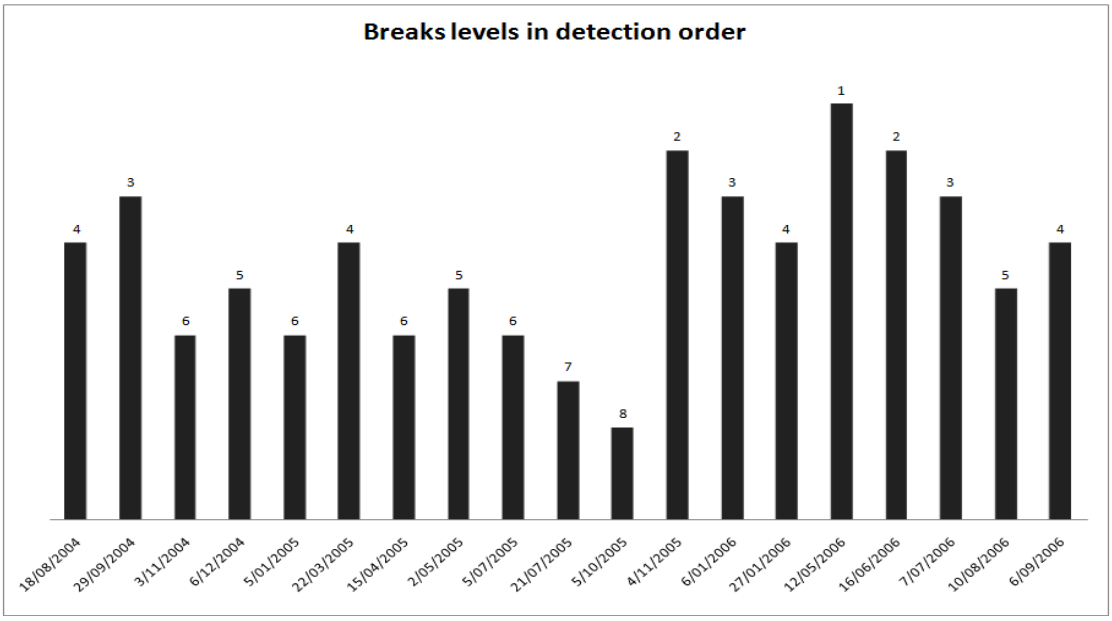

Figure 6.1: Levels of Breaks in Detection Order. (Level 1 is the 1st Break Date Found on May 12th, 2006)

the 545940th observation (05/12/06) the highest breakpoint statistic is obtained. The detection process continues until test statistics reach to the filtering criteria. The statistics for the full detection process of all 19 breaks are given in Table 6.4 in the Appendix.

Summary statistics of the raw data for the 20 sub-periods (19 breaks) are shown in Table 6.2. Over the 20 subperiods, the number of observations vary from 14566 to 71662, and the average durations vary from 0.61 to 1.31. Sub-period 8 (04/16/05-05/02/05) has the lowest mean and standard deviation of 0.6650 and 1.3002 while the highest mean occurred in sub-period 13 (11/05/05-01/06/06) at 1.3131. The adjusted volume is generally larger than 1 after sub-period 16 (05/13/06-06/16/06). Note the average mean adjusted duration and adjusted volume for the whole sample raw data sample is around 1.

A separate Weibull ACD (1,1) model is fitted on each of the 20 sub-periods and the parameters are shown in Table 6.3. Some of the parameters are quite different across different regimes. The constant parameter ω_0 in 20 sub-periods

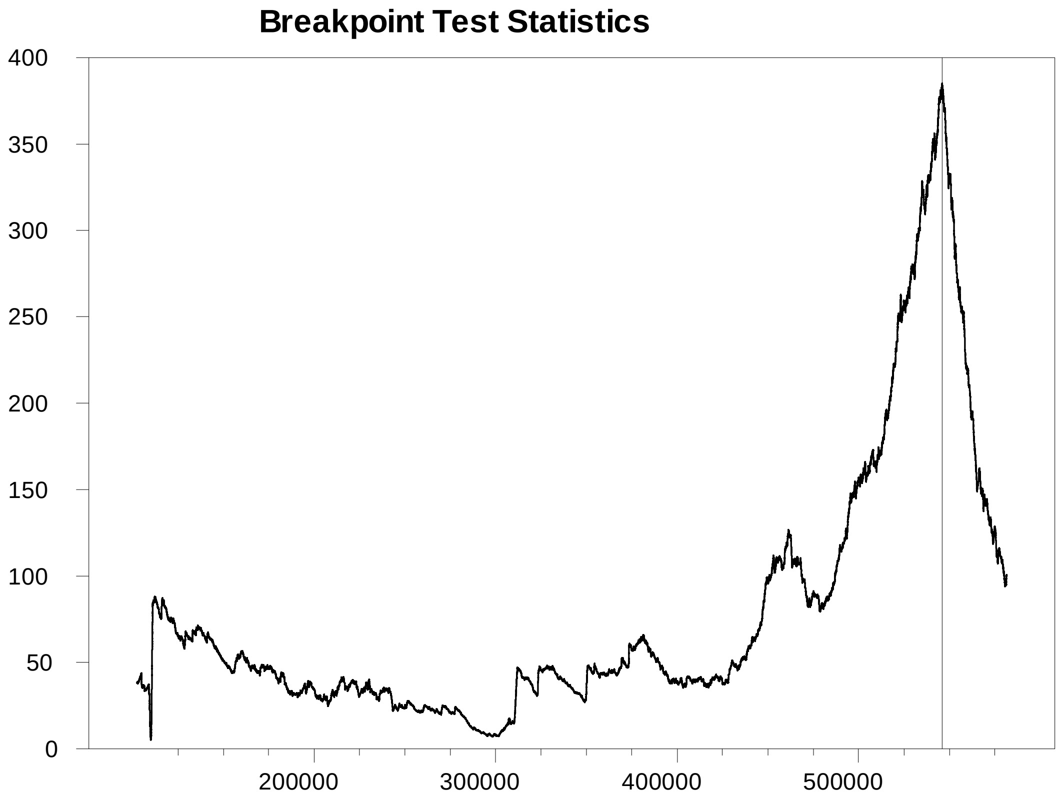

Figure 6.2: Lcation of the Break for the First Run on the Whole Data, Where x-axis is the Number of Observation and This Break Occurs on May 12th, 2006.

varies from 0.0058 to 0.0835. A smaller range is evident for the lagged duration parameters, changing from 0.0674 up to 0.1261. Impacts from past durations are evidently stronger towards to the end of the period in 2006. The estimated coefficients show $\gamma_1 + \omega_1$ are all very close to 1, indicating that there is a high level of persistence in the adjusted durations. The estimates of α in the standardized Weibull distribution are around 0.6 to 0.7 across all periods, indicating the conditional hazard function is decreasing at a moderate rate.[7] The volume parameter changes significantly from a minimum of -0.0003 in sub-period 10 to a maximum of -0.0042 in sub-periods 5 and 13, the negative impacts from past volume are 14 times stronger in the latter periods. Note that in Table 6.3, the conditional expected duration $\omega_0/(1 - \gamma_1 - \omega_1)$ is 0.72 and 0.66 in sub-periods 8

[7]Information on Weibull distribution and hazard function is presented chapter 4.

Table 6.2: Summary Statistics
of Individual Sub-periods

sub-period	NumObs	Mean duration	SD	adjusted volume
1: (07/01/04-08/18/04)	39999	0.9851	1.7791	0.9219
2: (08/19/04-09/29/04)	29762	1.1178	2.0960	0.9014
3: (09/30/04-11/03/04)	41048	0.7556	1.3770	0.9515
4: (11/04/04-12/06/04)	23131	0.9645	1.8091	0.9242
5: (12/07/04-01/05/05)	16899	1.3109	2.5452	0.8964
6: (01/06/05-03/22/05)	58613	1.0431	1.9457	0.9551
7: (03/23/25-04/15/05)	17953	0.9779	1.866	0.9626
8: (04/16/05-05/02/05)	17756	0.6650	1.3002	1.0543
9: (05/03/05-07/05/05)	44303	1.1570	2.1437	0.9488
10: (07/06/05-07/21/05)	14566	0.8844	1.6958	1.1288
11: (07/22/05-10/05/05)	54773	1.1038	1.9846	0.9775
12: (10/06/05-11/04/05)	31606	0.8048	1.4348	1.1139
13: (11/05/05-01/06/06)	35947	1.3131	2.4432	0.9597
14: (01/07/06-01/27/06)	17086	0.9133	1.8228	0.9956
15: (01/28/06-05/12/06)	71662	1.1784	2.1112	0.9936
16: (05/13/06-06/16/06)	46591	0.6067	1.877	1.1727
17: (06/17/06-07/07/06)	16191	0.9792	1.9551	1.0307
18: (07/08/06-08/10/06)	31687	0.8649	1.5358	1.1084
19: (08/11/06-09/06/06)	15323	1.2425	2.4661	1.0484
20: (09/07/06-09/29/06)	18737	0.9867	1.7701	1.0314

and 16. The sample mean in the raw durations in the corresponding periods are 0.66 and 0.60 respectively for the two periods from Table 6.3, thus it is reasonable that the conditional expected durations in these period are less than one.

Table 6.3: Parameter Estimates of Individual Periods from WACD(1,1)

	ω_0		γ_1		ω_1		α		v_1		$\omega_0/(1-\gamma_1-\omega_1)$
sub-period	estimate	t-stat	estimate	t-stat	estimate	t-stat	estimate	t-stat	estimate	t-stat	-
1	0.0406	18.9038	0.0933	30.6882	0.8727	222.2000	0.6699	215.9817	-0.0028	-25.6526	1.1942
2	0.0627	17.8824	0.1105	26.3848	0.8424	152.7772	0.6456	185.4522	-0.0039	-23.6386	1.3312
3	0.0058	10.5317	0.0674	35.3000	0.9290	505.2130	0.6981	223.0811	-0.0007	-8.3218	1.6111
4	0.0393	14.7641	0.0880	22.3217	0.8799	176.4371	0.6614	164.7926	-0.0033	-15.4162	1.2243
5	0.0543	11.8471	0.0997	19.8050	0.8668	139.8562	0.6341	136.8788	-0.0042	-6.5401	1.6209
6	0.0447	23.3923	0.1041	37.4155	0.8574	240.8052	0.6605	261.6420	-0.0018	-29.7093	1.1610
7	0.0368	12.3349	0.0988	21.0215	0.8678	148.3704	0.6690	148.0036	-0.0017	-13.1437	1.1018
8	0.0112	10.8433	0.0809	25.1116	0.9036	253.7145	0.7380	159.5278	-0.0004	-8.4402	0.7226
9	0.0602	22.1011	0.1049	31.8197	0.8519	200.3624	0.6517	227.8147	-0.0036	-24.3014	1.3935
10	0.0054	6.8150	0.0806	24.5471	0.9154	298.7944	0.7033	152.3703	-0.0003	-5.2888	1.3500
11	0.0373	20.1095	0.0954	35.5770	0.8777	270.2432	0.6520	254.1141	-0.0028	-23.6622	1.3866
12	0.0311	15.6009	0.0827	25.9912	0.8829	202.4560	0.6808	193.7455	-0.0013	-14.1988	0.9041
13	0.0835	19.8132	0.1205	28.9896	0.8249	148.0283	0.6436	216.0052	-0.0042	-16.6792	1.5293
14	0.0153	9.6609	0.0735	21.6783	0.9176	257.4435	0.6879	150.6308	-0.0021	-8.4882	1.7191
15	0.0615	24.3764	0.1155	39.9994	0.8359	214.6281	0.6504	288.7600	-0.0015	-29.5907	1.2654
16	0.0194	20.0199	0.0909	35.6317	0.8798	276.1484	0.7176	243.1682	-0.0007	-11.9745	0.6621
17	0.0553	14.2023	0.1197	21.0623	0.8234	106.6973	0.6666	146.8735	-0.0011	-10.1462	0.9367
18	0.0337	18.1399	0.1156	30.0979	0.8495	187.9215	0.6849	195.1508	-0.0010	-15.0314	0.9659
19	0.0519	12.8510	0.1261	21.6255	0.8387	126.6092	0.6476	137.9711	-0.0024	-8.7993	1.4744
20	0.0327	11.9293	0.1213	24.9747	0.8532	159.3299	0.6883	151.4670	-0.0018	-4.4097	1.2824

The individual WACD model parameters are plotted to assist further analysis. Changes in ω_0 and α parameters are all within an reasonable range and there is no obvious trend. The parameter plots are in Figures 6.5, 6.6, 6.7, and 6.8 in the Appendix. The past conditional expected duration parameter ω_1 experience a slow downward trend as shown in Figure 6.6. This pattern indicates that weaker impacts arises from expected durations along the time line. The past durations parameter γ_1, changes almost symmetrically with parameter ω_1, with a slightly upward trend as shown in Figure 6.7. Note that the distributional shape parameters α do not fluctuate very much, indicating the sample duration distribution form remains in a similar form during the sample period examined. However, the distributional shape may change sometime if sample period examined is more volatile. This matter is addressed in the following chapter where a more volatile crisis period sample data is examined. Interestingly the only parameter which varies significantly is from volume. Hence analysis based on volume parameter v changes is primarily focused in this chapter.

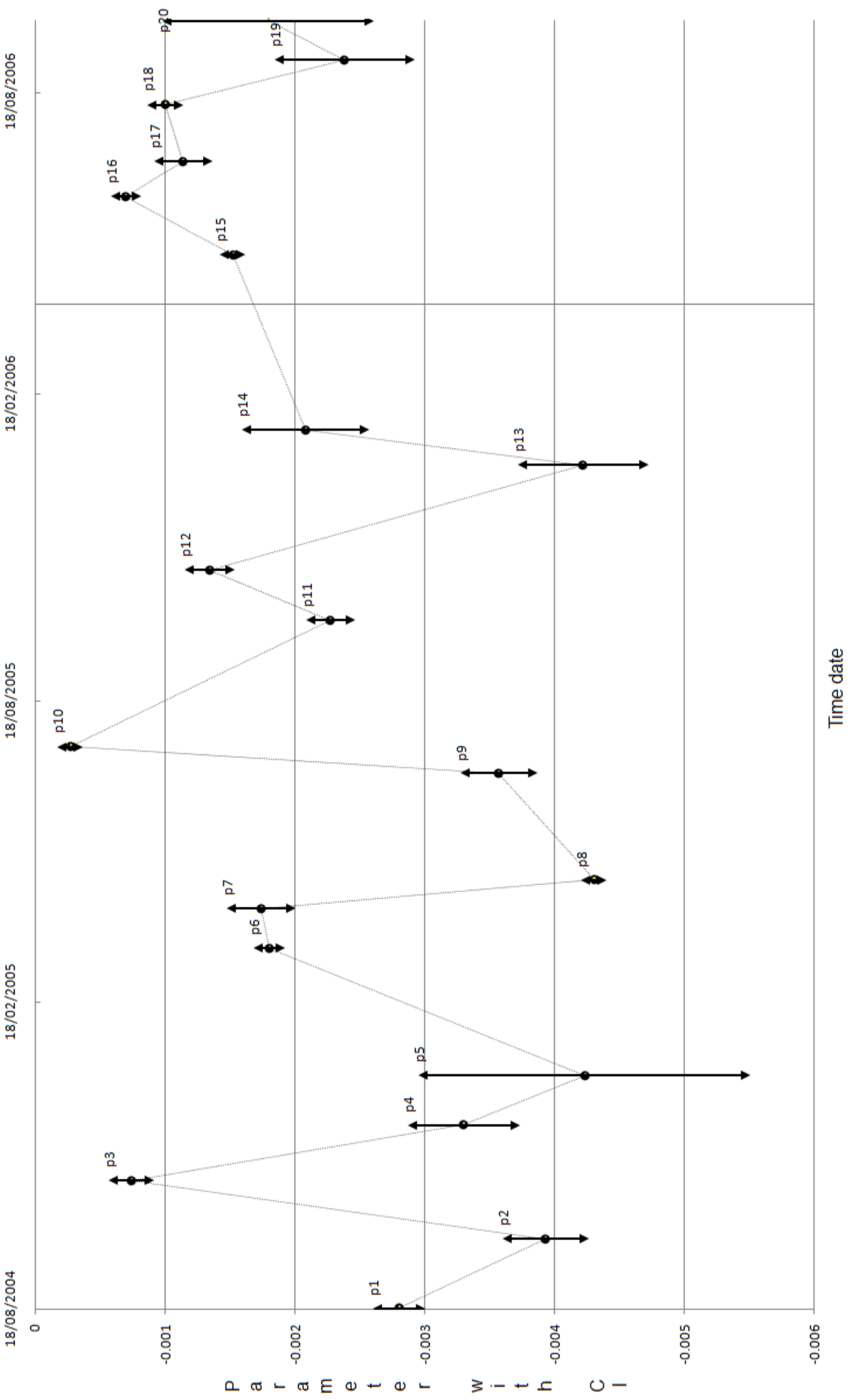

Figure 6.3: Estimated Parameters v_1 with ± 2 Standard Error Confidence Interval for 20 Sub-periods of S&P 500 Duration Data.

The volume parameters for all 20 individual sub-periods are graphed in Figure 6.3. The vertical grid line represents the most significant break which occurred on 12th May, 2006. As can be seen the magnitude of the negative coefficient becomes smaller immediately following the most significant break point. Before 12th May, 2006, the volume parameters seems to fall into two regimes, with the first regime in the range from -0.0003 to -0.0023, and second regime from -0.0028 to -0.0042. However, immediately following the most significant break point, the parameter v has contracted with small standard errors. This could be caused by the large increase in number of observations in these sub-periods. After 3 more sub-periods, event effects start to fade and volume coefficient starts to climb back. The cause of this most significant break point is possibly the bankruptcy of the Washington based Merit Financial Inc. took place in early May, 2006. The merger plan between CBOT and CME was only announced in October, 2006, whereas the data examined in this chapter ends in September, and the most significant break is detected in May, 2006. Apart from the most significant break found on 12th May, 2006, it is very difficult at this stage to interpret this structural change in relation to the market. However, in the following chapter, more analysis can be drawn based on structural changes since the data sample continues through the global financial crisis period where many economic events occurred.

The Weibull ACD(1,1) still does not fit some of the sub-periods very well. Some further studies such as fitting a more complicated ACD model may help to remove the nonlinear pattern of the data. For example Zhang et al. (2001) fit a three-regime Threshold ACD model while detecting the structural breaks. Also if we relax the filter for breaks and allow more break detections, the model may fit better.[8]

6.6 Conclusions

In this chapter, we performed Andrews Ploberger structural break tests on a Weibull ACD model using after-hour electronic futures market data set spanning two years. The sample is divided into 20 sub-periods from 07/01/04 to 09/29/06 using the AP test. The parameters change significantly across different sub-periods, especially for volume parameters. Although the Weibull ACD (1,1) model still does not fit some of the sub-periods very well and some remaining nonlinearity is evident due to the restriction of the filters and model selection, the overall model fitting improves by picking up all the sub-periods. A more complex ACD model with higher lag orders could be fitted into the structural break detection in the future studies.

In next chapter, testing structural breaks within ACD model is applied to S&P 500 data during the 2007-2008 global financial crisis period. This new data sample is expected to have larger degree of structural effects given that many economic events took place during the sample period, potentially allowing duration dynamics to be better interpreted as aligning with these economic events.

[8] We also applied the AP breaks test on the residual from a Weilbull ACD(1,1) model by treating them as an AR(1) process. If the ACD model is correctly estimated and there are no structural breaks in the data, the residuals should not be serially correlated. Three breaks are located and presented in Table 6.5 in Appendix.

6.7 Appendix

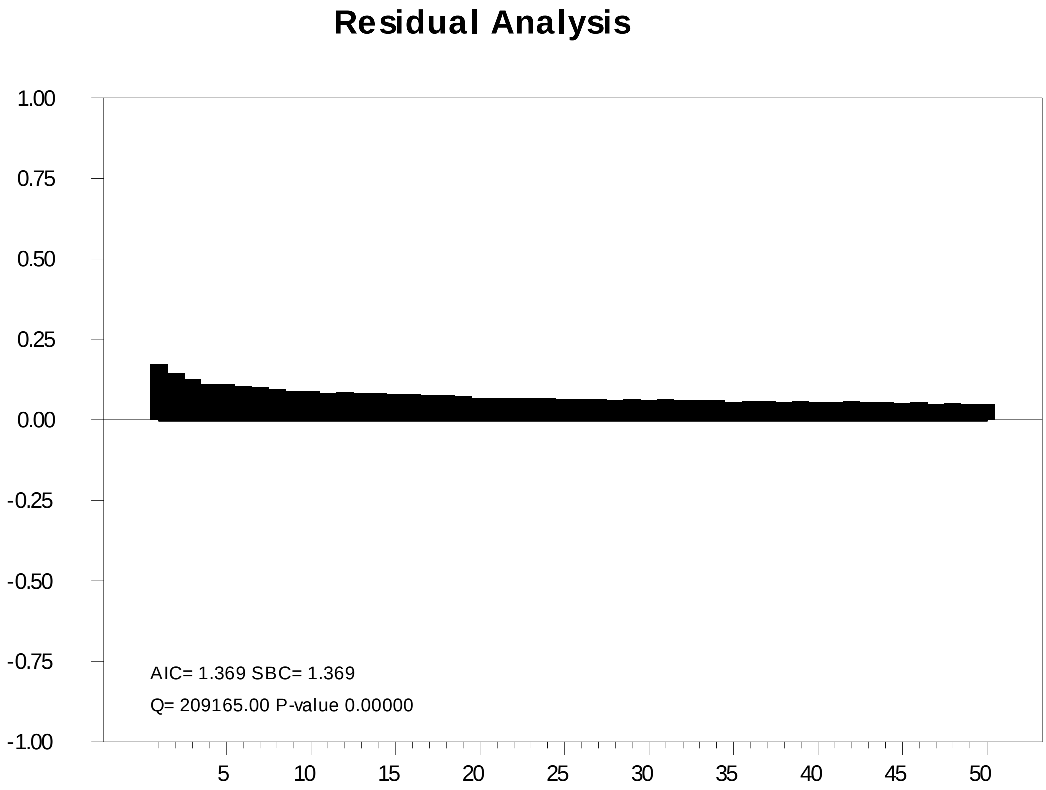

Figure 6.4: ACF with 50 lags for S&P Raw Data

Table 6.4: Subperiods AveLM and SupLM Test Statistics(July 04-Sep. 06)

Break Date	Break Level	$SupLM$	p-value	$AveLM$	p-value
18/08/04	4	83.6973	0.000	35.7033	0.000
29/09/04	3	103.6969	0.000	46.1897	0.000
03/11/04	6	103.6545	0.000	97.4823	0.000
06/12/04	5	40.705524	0.000	15.6366	0.000
05/01/05	6	78.8267	0.000	34.2424	0.000
22/03/05	4	143.6403	0.000	66.5692	0.000
15/04/05	6	237.5907	0.000	113.5759	0.000
02/05/05	5	234.9926	0.000	110.3186	0.000
05/07/05	6	143.4637	0.000	66.3300	0.000
21/07/05	7	408.7457	0.000	197.6461	0.000
05/10/05	8	137.9263	0.000	62.8841	0.000
04/11/05	2	239.0274	0.000	112.1628	0.000
06/01/06	3	76.9878	0.000	32.4823	0.000
27/01/06	4	267.0580	0.000	126.57462	0.000
12/05/06	1	385.0941	0.000	185.3365	0.000
16/06/06	2	362.1198	0.000	174.0727	0.000
07/07/06	3	65.0091	0.000	27.6672	0.000
10/08/06	5	35.7799	0.000	13.5921	0.000
06/09/06	4	81.0641	0.000	35.5097	0.000

Table 6.5: Three Sub-periods from Testing the Model Residuals

sub-periods
1: (07/01/04-07/08/05)
2: (07/09/05-06/14/06)
3: (06/15/06-09/29/06)

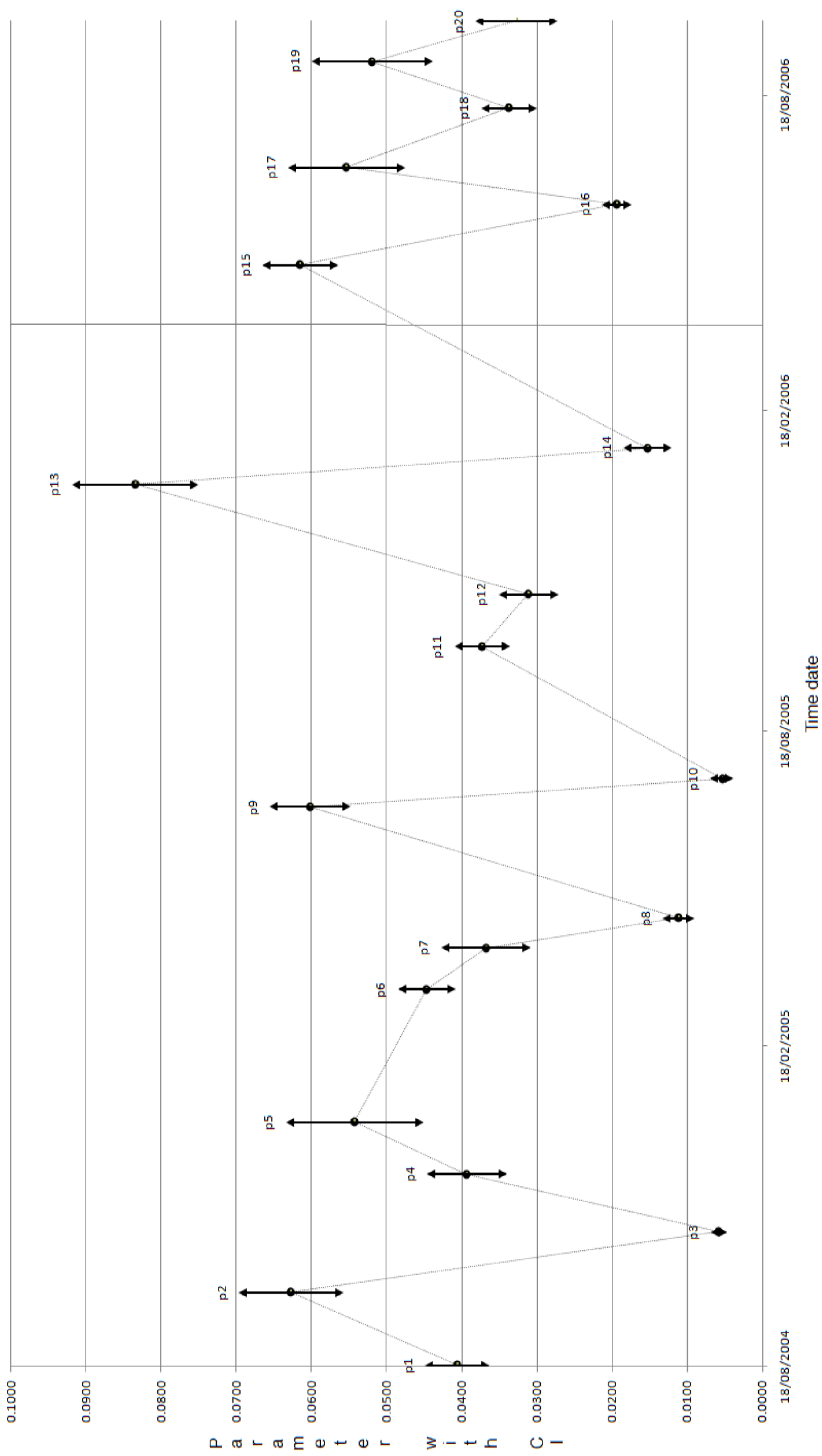

Figure 6.5: Estimated Parameters ω_0 with ± 2 Standard Error Confidence Interval for 20 Sub-periods of S&P 500 Duration Data. (with vertical grid line symbolises the most significant break date)

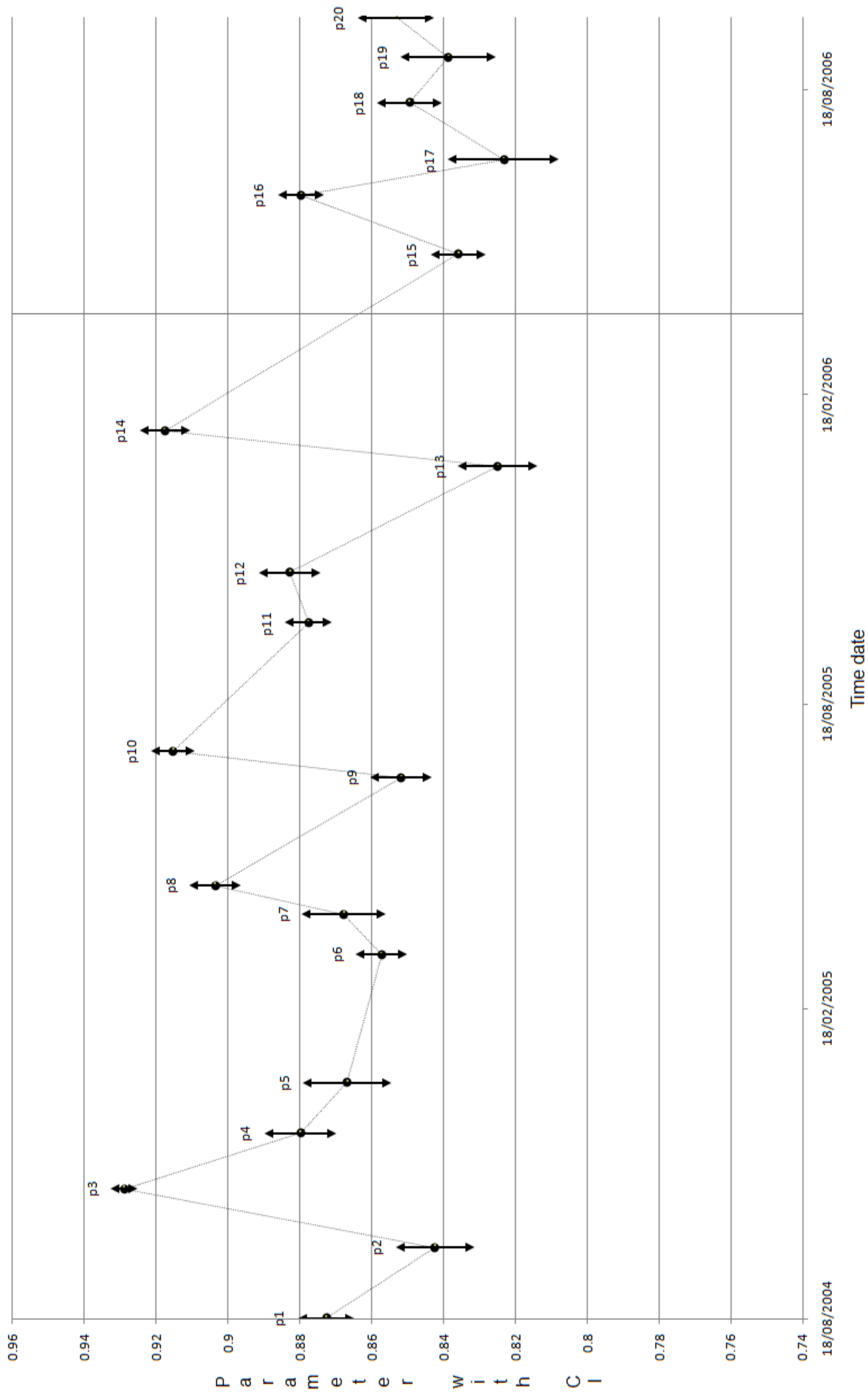

Figure 6.6: Estimated Parameters ω_1 with ± 2 Standard Error Confidence Interval for 20 Sub-periods of S&P 500 Duration Data.

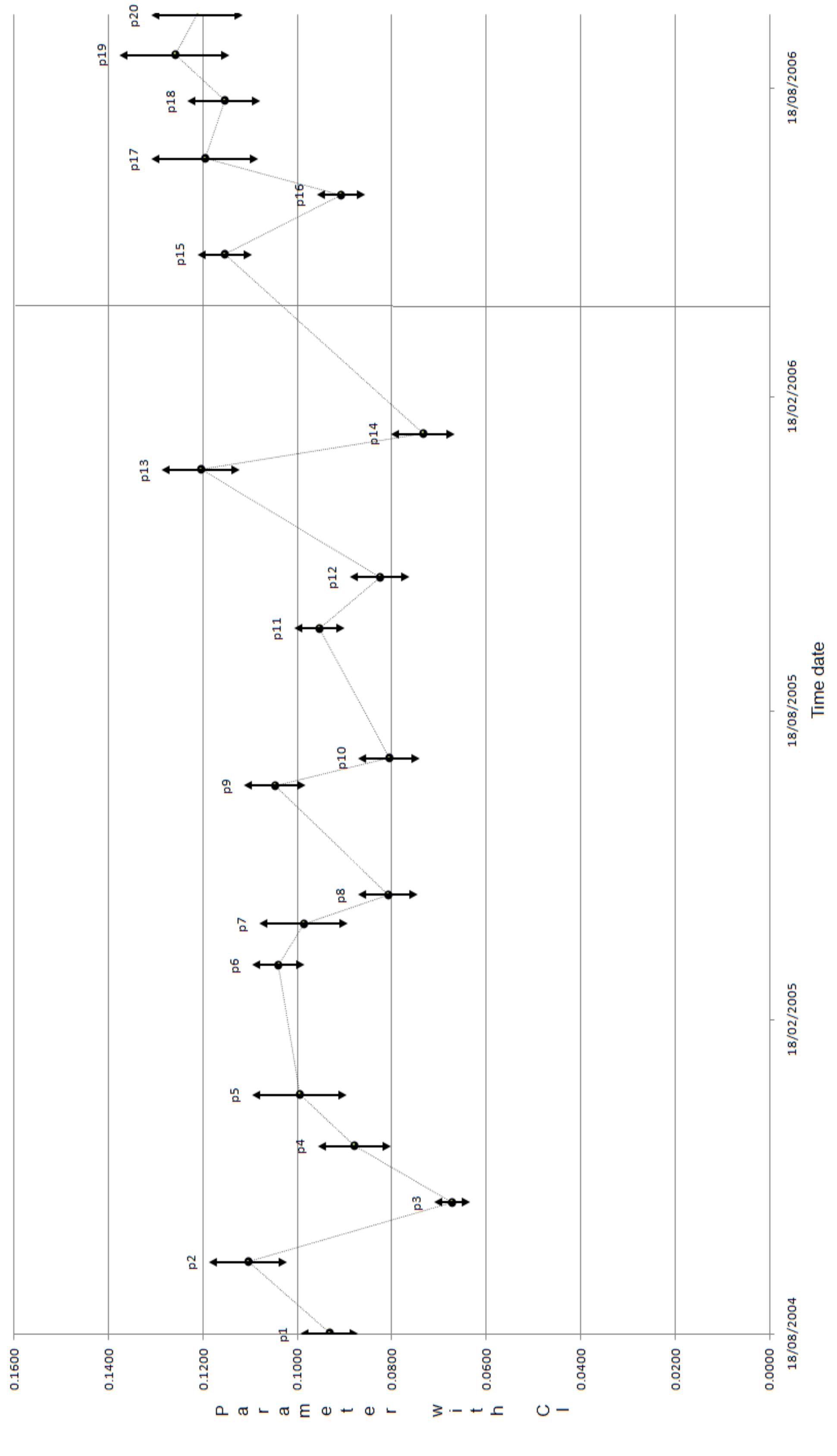

Figure 6.7: Estimated Parameters γ_1 with ± 2 Standard Error Confidence Interval for 20 Sub-periods of S&P 500 Duration Data.

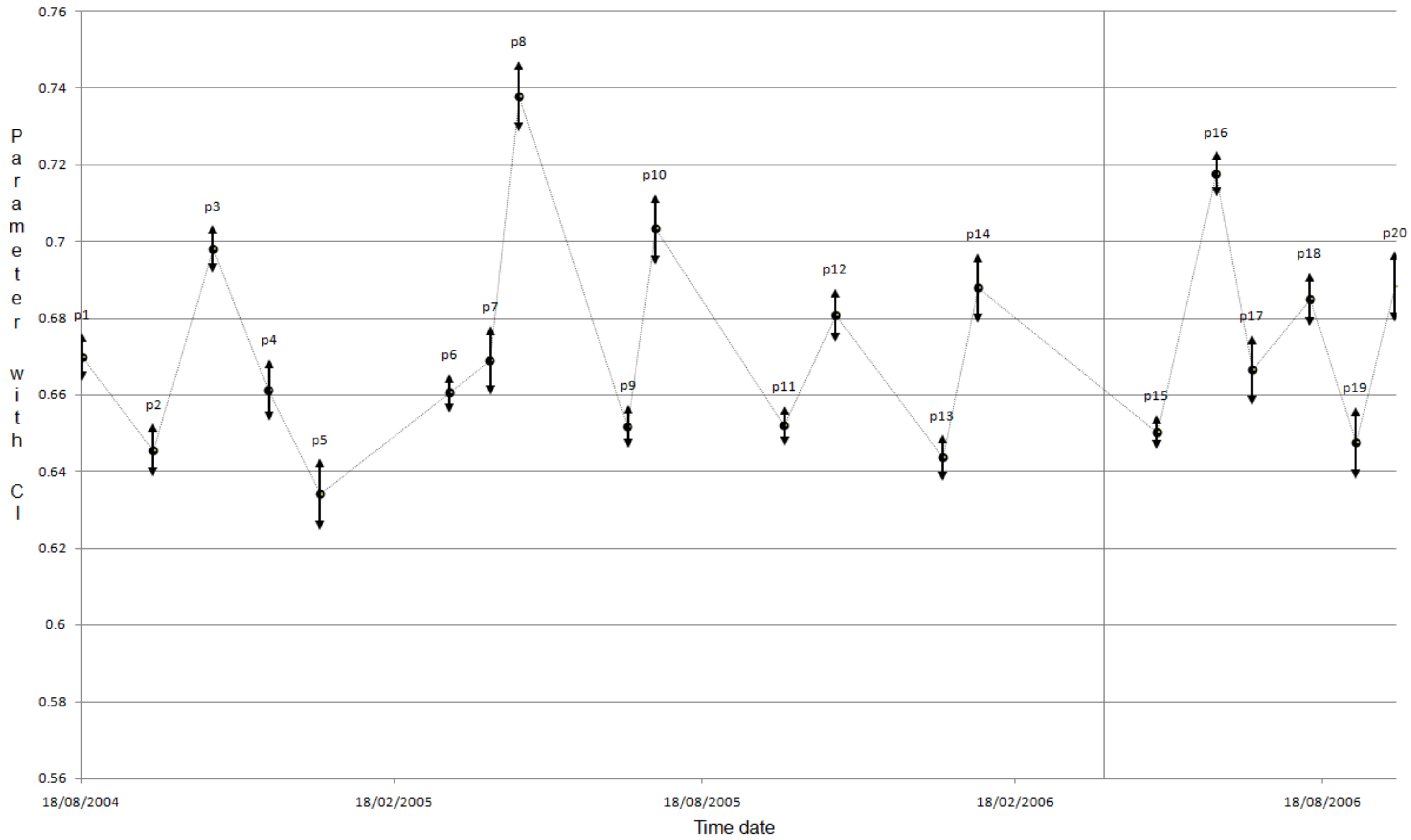

Figure 6.8: Estimated Parameters α with ± 2 Standard Error Confidence Interval for 20 Sub-periods of S&P 500 Duration Data.

Chapter 7

ACD Modelling with Structural Breaks (Crisis Period)

7.1 Introduction

In this chapter the study of potential structural breaks in ACD models is extended to a comprehensive investigation of durations data around the 2007-2008 global financial subprime crisis period.

The subprime crisis of 2007 is recognized as the worst financial crisis since the 1930s Great Depression. Worldwide financial institutions, banks, and stock markets suffered severely and economic activity declined significantly. The details of crisis impacts were introduced in chapter 3. Relevant literature on the crisis includes Blackburn (2008), Kolb (2010), Ely (2009), Liebowitz (2009), and Reinhart and Rogoff (2008). In this severe time period, significant structural effects are highly likely due to the economic events taking place. While the ACD class of models are able to capture the temporal dependence of the clustering duration process within a short period, the question of whether ACD models are adequate for modelling longer periods of data, especially during extremely volatile market movements, remains uncovered.

The crisis period data provides a rich sources of price, volume, and investor behavioural changes, producing an opportunity to study duration changes together with other marks in market. Duration shifts are often associated with news events and structural changes, and it is of interest to see whether these are aligned. WACD models are fitted to the data sample for the period from the beginning of October, 2006 to the end of December, 2008, to study the ACD structure through extreme volatile market changes and potential multiple structural changes. Popular approaches available for testing unknown structural breaks in conditional models include the Sup-LM based tests in Andrews (1993) and Andrews and Ploberger (1994); CUSUM test from Inclan and Tiao (1992) and multiple structural break test in Bai and Perron (1998). In this chapter we adopt the Sup-LM based tests as in chapter 6, to locate the structural breaks from the model and break up the data sample into sub-periods. A basic Weibull ACD(1,1) model is fitted in each individual sub-period.

The application in this chapter follows the earlier chapters in being based on the after-hours futures market traded through the Global Exchange (GLOBEX) platform. However the sample is now based on crisis period and is for 2 later years than the earlier part of the thesis. The volume in this market has grown significantly. In fact, electronic market trading from the Chicago Mercantile Exchange (CME) takes around 80% of total market volume traded, and electronic trading is generally seen as the way of future. The after-hours market provides trading after the clock and records transaction from all over the world. For these reasons, the data in the crisis period in the electronic market can be treated as a relatively complete documentation of investors' trading behaviour during the global crisis, embedded with many uncovered stories.

The chapter is constructed as follows: Section 2 introduces the after-hours market and the financial crisis; Section 3 provides the data description; Section 4 outlines the methodology, Section 5 includes the estimation results and finally Section 6 concludes.

7.2 The After-hours Market During The Global Financial Crisis

The 2007-2008 Global Financial Crisis (GFC) has been generally considered as the worst financial crisis since 1930s. The S&P 500 data sample examined in this chapter starts from 2006, when the U.S. interest rate started to rise again due to the pressure of a falling US dollar. Rising defaults in mortgages and shrinking housing values started to cause damage; first apparent in hedge funds. In late 2006, fears of losses in higher tranches of asset backed securities started to grow. Consequently the U.S. stock market was bearish from the end of 2006 till early 2007.

In mid 2007, the build up of increasing fear of defaults created a credit crunch, investors started to lose confidence in the value of sub-prime mortgages. Tighter inter-bank lending made it difficult for suffering companies to recover. Market investors lost confidence in sub-prime mortgage values, leading to huge level of trading to adjust portfolios.

The crisis started to strike the market following the collapse of two Bear Stearns hedge funds on 17th, July 2007. Foreign investment banks suspended investment funds in the subprime mortgage market, and the U.S. Federal Reserve intended to cut the discount lending rate and agreed to lend directly to Wall street firms. Unfortunately the market deteriorated further and more firms collapsed during the year 2008. In September 2008, a series of economic events took place. Followed by the takeover of Fannie Mae and Freddie Mac, Merrill Lynch was sold to Bank of America, and Lehman Brothers collapsed. American International Group survived bankruptcy due to a $85 billion loan facilitated by the Federal Reserve.

Since global financial markets are highly integrated, the sudden rise in US financial market volatility and preference for liquidity was quickly distributed worldwide. Many leading banks and large financial institutions were affected due to the collapse in the value of mortgage-based securities in US and Europe during 2007 to 2008.

The details of futures contracts traded on GLOBEX in the after-hours are not introduced here again (see more details in chapter 3). Around the 2007 global financial crisis period, the volume traded in the electronic market increased dramatically. Figure 7.1 suggests that more transactions took place in the 2006-08 period, compared with the former two-year (2004-06) period examined in Chapters 4, 5, and 6. A merger between CME group and the Chicago Board of Trade (CBOT) took place in late 2006, and all the electronic products merged in January 2007. This is responsible to some degree for the huge (67%) growth in total volume traded in the CME group, and 82% growth in GLOBEX from the 2006 to 2007 financial year[1]. Also, from January 2008, the U.S. stock market experienced a major downturn. With the presence of over-night and international

[1] The statistics are collected and calculated from CME group annual reports 2002 to 2009.

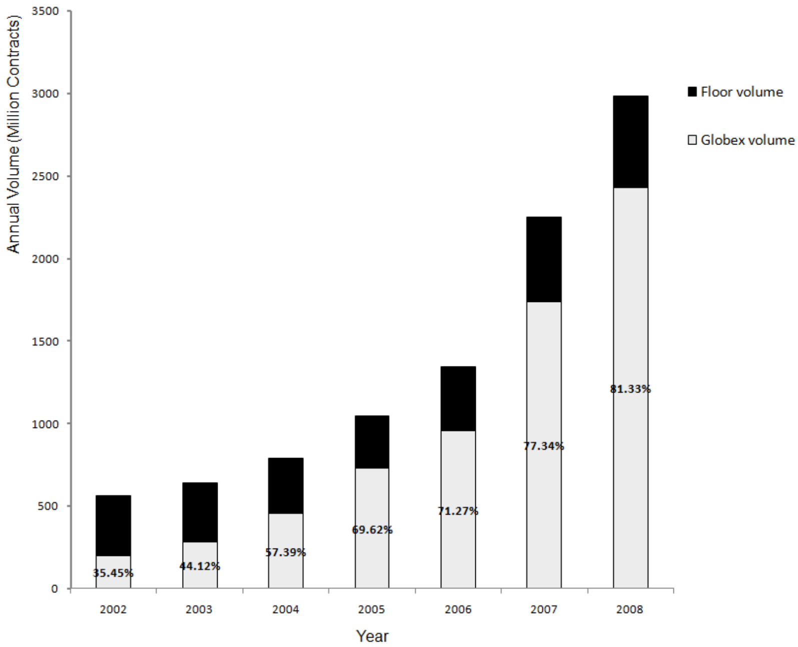

Figure 7.1: GLOBEX Yearly Trading Volume Percentage Growth

news events, investors may prefer to access GLOBEX to adjust their positions rather than wait until the next day for the floor market to open. According to statistics from CME web site, the volume traded in the whole year of 2008 for all markets was 2.98 billion contracts, and volume traded on GLOBEX was 2.43 billion contracts, taking almost 82% of the whole market. Not only have more transactions occurred within U.S., there have been growing trade from all over the world. In 2009, 19% of volume traded in the electronic market was transacted outside U.S. trading zone, compared with 17% in 2008 and 9% in 2007.

7.3 Data Description and Methodology

The data sample used in this chapter is the standard equity futures contracts for the S&P 500 traded in the after-hours electronic market on the GLOBEX

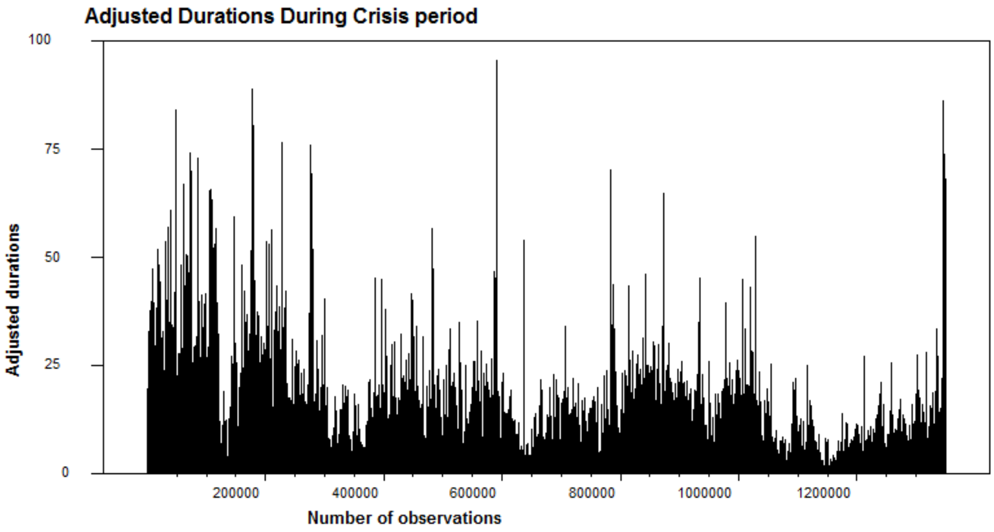

Figure 7.2: Time Series Plot of Adjusted Durations for S&P 500 During Crisis Period

exchange from beginning of October 2006 to the end of December 2008[2]. The sample period is selected to include the global financial crisis of 2007 to end of 2008. The duration data in this chapter are first diurnally adjusted using a spline to remove the daily pattern as in Chapter 6. The adjusted duration time series data for the S&P 500 have 1,349,514 observations over the sample period. A time series plot of the adjusted durations is shown in Figure 7.2.

Apart from duration, volume is also considered in the model as an additional mark. Volume is also diurnally adjusted to remove the daily seasonality. All the transactions occurring at the same time are aggregated to avoid zero durations. The diurnal patterns for duration and volume are shown in panel (a) and (b) in Figure 7.3, where the horizontal axis represents the time of a day in the form of number of seconds from mid-night, and vertical axis is the adjusted diurnal series.

The duration and volume diurnal patterns during pre-crisis and crisis period

[2]The data sample in this chapter is a continuation of the data (July 2004-September 2006) examined in previous chapter.

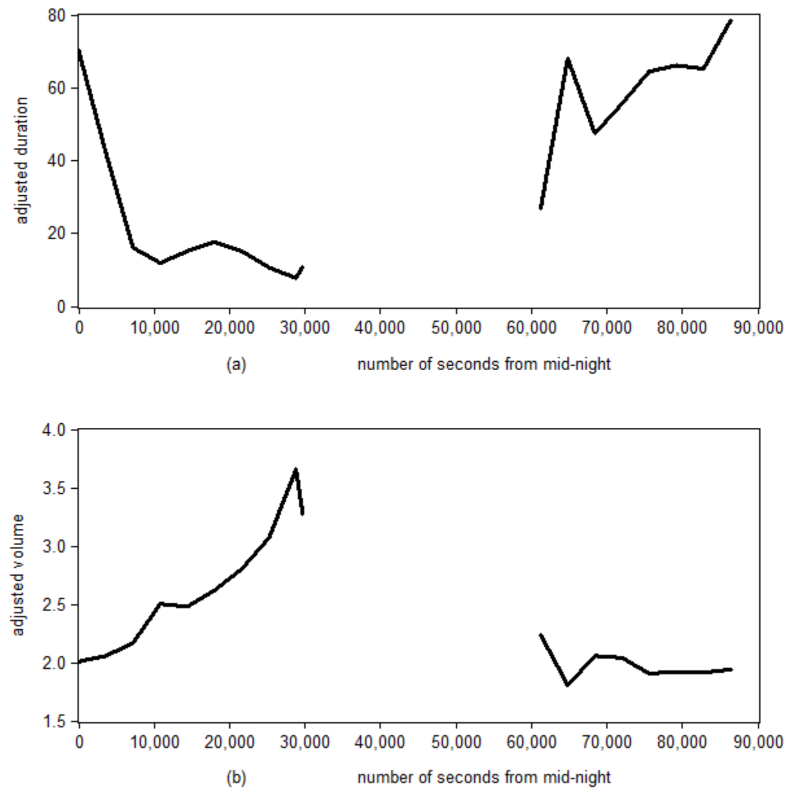

Figure 7.3: Diurnal (a) Duration and (b) Volume Pattern for S&P 500 During Crisis Period

are shown in Figure 7.3 and 7.4[3]. The diurnal patterns for the crisis period have clearly changed from the pre-crisis data. The differences are clear for both adjusted duration and volume between mid-night and 8:30 am. Compared to the 2004-2006, the adjusted duration during crisis period drops and volume rises more quickly in the early morning before 8:30 am. This indicates that the trading activities were more intensive, and investors acted more quickly according to their available information during the morning period, which is consistent with Dungey, Fakhrutdinova, and Goodhart (2009). The quickly rising diurnal adjusted volume is consistent with this view.

The range of diurnal adjusted duration is between 10 and 80 seconds during the crisis period, compared to from 10 to 170 seconds before the crisis period,

[3]2004-2006 diurnal pattern is from Figure 4.3 in chapter 4.

indicating the average level of waiting time during the crisis is much shorter. However, the range of diurnal adjusted volume has actually decreased during the crisis. This might due to less clustering of trading during crisis period. In other words, compared with non-crisis period, the trading activities during the crisis period are more evenly distributed, but with shorter waiting time. Clustering behaviour is also discussed later in this chapter in association with changes in ACD model parameters.

From the comparison between Figure 7.3 and 7.4, it is clear that the adjusted volume pattern has shifted downwards overall during the crisis period. As the time period between mid-night and 8:30 am mostly corresponds trading activities from Europe, it is a good indication that worldwide investors trading behaviour has changed due to the global financial crisis.

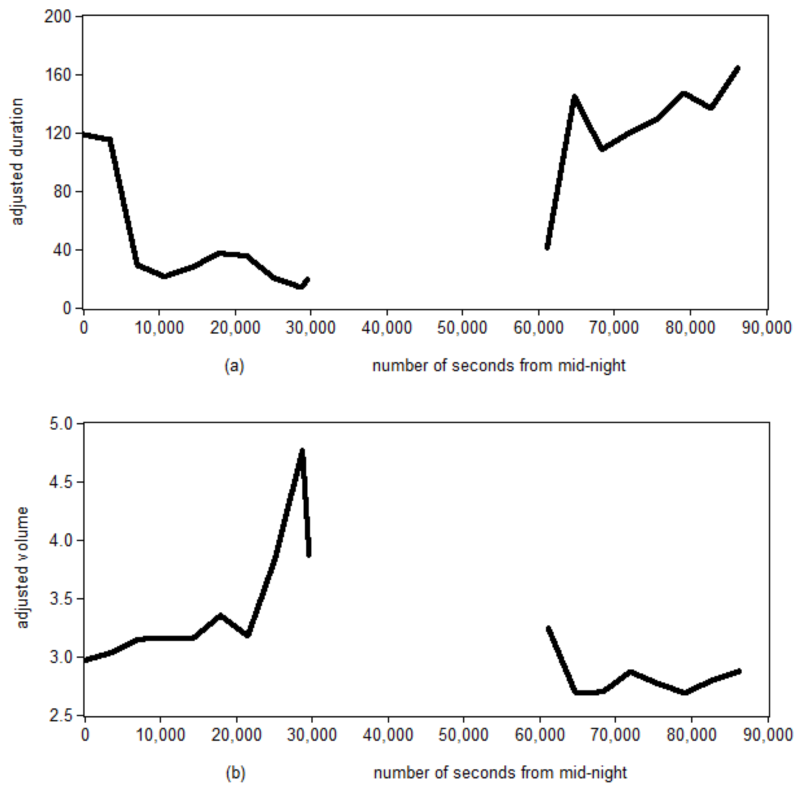

Figure 7.4: Diurnal (a) Duration and (b) Volume Pattern for S&P 500 During 2004 to 2006 Period.

The LM-based tests of Andrews (1993) and Andrews and Ploberger (1994) are adopted for detecting structural breaks in the ACD model of this chapter. The LM-based tests treat the multiple change points detection as an extension of the single change point problem as described in chapter 6. The sample data are first fitted with a Weibull ACD (1, 1) model. The Weibull ACD (1, 1) model used in this chapter follows:

$$\begin{cases} X_i = \psi_i \varepsilon_i \\ \psi_i = \omega_0 + \sum_{j=0}^{m} \gamma_j x_{i-j} + \sum_{j=0}^{q} \omega_j \psi_{i-j} + v_i vol, \end{cases} \quad \varepsilon_i \sim i.i.d. \tag{7.1}$$

where ω, γ_j and ω_k are parameters, and p and q represent the lag orders. The adjusted volume is added as an additional mark in the model, with a parameter v_1. Manganelli (2005) introduced a vector autoregression approach to model duration and volume simultaneously, producing greater feedback on volatility. However, this thesis does not pursue this approach as structural effects during GFC period is our main focus. The Andrews and Ploberger (AP) tests are then applied to the derivative series at the maximizing ACD parameters. The date which generates the largest AP LM statistics is recognized as the most significant break. Two separate Weibull ACD (1,1) models are again applied on the two sub-periods divided by the break as before. The new ACD model parameter derivative series are then used for succeeding round of the AP test. The same process continues until no further break points are located. Details of the Andrews and Ploberger LM-based tests have been covered in chapter 6, and the methodology of Weibull ACD model is also explained in chapter 4, and are therefore not repeated here.

7.4 Results

In this section, the results of structural break detection and ACD model estimates over individual sub-periods are presented. In order to limit the number of breaks, a filter is implemented. If the length of a particular sub-period is less than 22 days[4], we stop further testing for breaks. However, it is still possible for a shorter inter break period to emerge as the result of finding a break located near the end point of a previously defined sub-period. This is consistent with the filter adopted in structural break studies in chapter 6. In the following subsections, the general break test results, Weibull ACD model estimated results over the segmented sub-periods, and their relations to market events are presented.

7.4.1 Structural Breaks Detection Statistics

The most significant break over the period of 2006-08 occurred on 24th July, 2007, with the highest Andrews and Ploberger LM test statistics. This date is consistent with many chronologies of the crisis which place its beginning in mid-July, 2007. Figure 7.5 presents the hierarchy of the complete detection process. The first break is recorded with symbol A1 on 24th, July 2007. The detection process continues until break statistics are no longer significant at 1% level, or reaches the 22 days limit. The final sub-periods are recorded as dark shaded boxes in Figure 7.5. There are 30 breaks detected in total, and the S&P 500 duration data are segmented into 31 sub-periods. Almost 2/3 of all sub-periods occurred after the most significant break on 24th, July 2007.

To illustrate the individual detection process graphically, Figure 7.6 provides

[4] This comes from an average of 22 business days in a month.

the locations of breaks A1, B1, and B2 over their timeline of sample observations. The dates with highest $SubLM$ statistics are recognized as breaks in their particular sample range. The same detection process continues for the rest of the hierarchy in Figure 7.5. The calendar length of the data sample before break point A1 is 10 months, and after is 17 months. However, the break detection graph in Figure 7.6 clearly indicates a disproportionately larger number of observations after break point A1. In particular, around 81% of the sample data are recorded within the 19 months after the 24th July, 2007, whereas only 19% within the 10 months before 24th July, 2007. Clearly more frequent market structures took place after the trigger of the GFC.

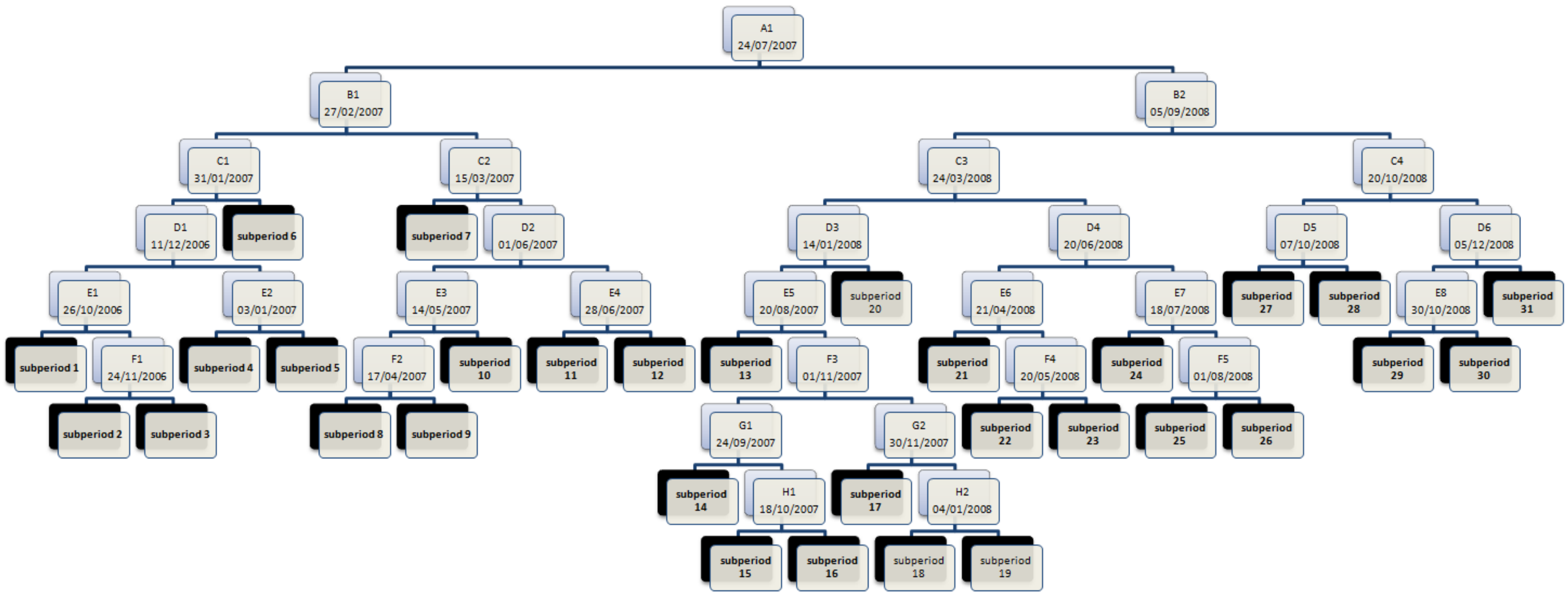

Figure 7.5: Diagram of Structure Break Test Procedures for the S&P 500 Trade Duration. The Parametric Model Used to Conduct the Test is the Weibull ACD (1, 1) model.

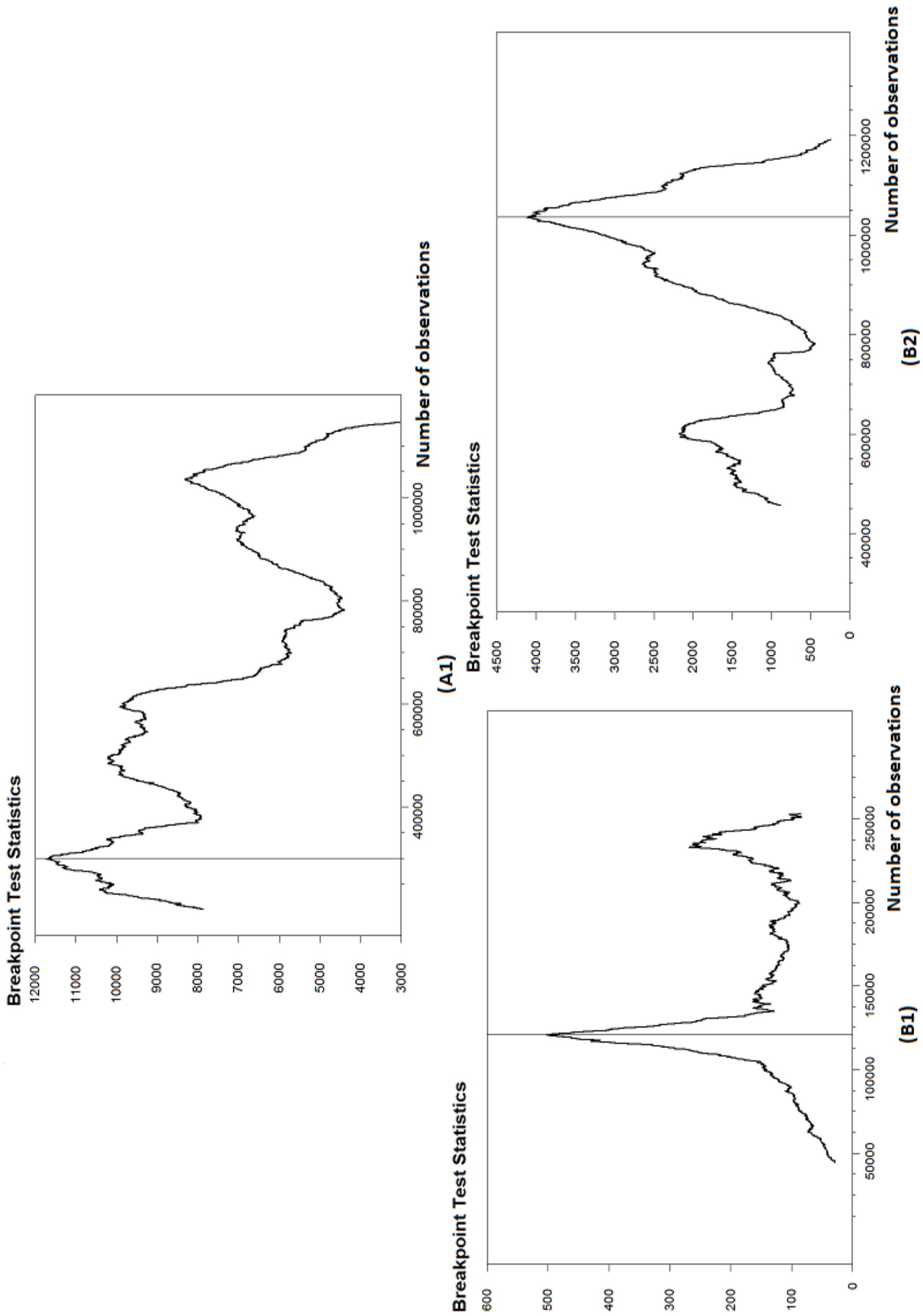

Figure 7.6: LM Statistics for the First 3 Detections for the S&P 500 Trade Duration, Over the Period of 02/10/2006 to 31/12/2008. Panel (A1) Represents the Detection of the Most Significant Break on July 24th, 2007. Panel (B1) and Panel (B2) Follow the Same Detection Process as in Figure 4.

Table 7.1: Break Levels, SupLM, and AveLM Statistics

Break points	date	*SupLM*	*AveLM*
A1	27/07/2007	11734.0390	5856.6780
B1	27/02/2007	502.4437	244.1396
B2	05/09/2008	4125.5584	2053.4134
C1	31/01/2007	422.1132	203.6206
C2	15/03/2007	879.9792	432.8807
C3	24/03/2008	823.5639	404.2477
C4	20/10/2008	1333.0450	659.0057
D1	11/12/2006	137.2361	62.0176
D2	01/06/2007	381.3638	138.5892
D3	14/01/2008	1928.7662	955.1089
D4	20/06/2008	307.1334	146.1626
D5	07/10/2008	443.6384	215.5278
D6	05/12/2008	436.0073	209.9600
E1	26/10/2006	66.0545	27.3587
E2	03/01/2007	71.1958	31.2462
E3	14/05/2007	28.8596	10.7311
E4	28/06/2007	119.7857	54.9419
E5	20/08/2007	1060.5757	522.0218
E6	21/04/2008	142.2736	64.9666
E7	18/07/2008	244.3920	115.8616
E8	30/10/2008	105.3707	46.7086
F1	24/11/2006	35.5652	13.0193
F2	17/04/2007	42.3944	16.5050
F3	01/11/2007	412.4973	199.4475
F4	20/05/2008	93.2994	41.0876
F5	01/08/2008	44.7753	17.7970
G1	24/09/2007	102.4862	45.1242
G2	30/11/2007	144.5467	66.1161
H1	18/10/2007	153.6428	70.8131
H2	04/01/2008	119.5095	54.0799

Table 7.1 presents the Andrews Ploberger *SupLM* and *AveLM* statistics for all the breaks detected in Figure 7.4. The *SupLM* and *AveLM* statistics are all significant at 1% level. Break point A1 yielded the largest AP LM statistic of 11734, and its average LM statistic is 5856, strongly indicating possible structural effects within the examined sample period. As we move through the hierarchy, the corresponding *SupLM* and *AveLM* statistics become smaller and smaller. At the bottom level of the detection hierarchy, the *AveLM* statistics for F1 and

F2 are 13 and 16 respectively.

The general statistics of 31 sub-periods divided by these 30 break points are listed in Table 7.2. The most significant break point A1 occurred between sub-period 12 and 13. As mentioned before, there are many more duration observations from sub-period 13 onwards, indicating an increased level of market participation after the major change occurred. The conditional and unconditional mean duration are relatively smaller after sub-period 13. Only 1 sub-period has an average duration less than 1 second in the pre-crisis[5] periods, compared with 10 sub-periods after the crisis was triggered. Short duration is an indication of more frequent trading and hence is an indication of increased liquidity in the market as we have controlled for volume. Standard deviations become smaller and skewness and kurtosis becomes larger on average after sub-period 13.

7.4.2 ACD Model Estimate Results Over Sub-periods

The Weibull ACD (1, 1) model parameters for all 31 sub-periods are tabulated in Table 7.3. These parameters, namely the constant term ω_0, past observed duration parameter γ_1, past expected duration parameter ω_1, volume parameter v_1, and Weibull distribution shape parameter α, from equation (7.1), are presented graphically from Figure 7.6 to Figure 7.10. A ± 2 standard error confidence interval is inserted in each plot. The interpretation of these parameter changes are presented in the following subsections.

[5]From here onwards, we treat periods before 24th, July 2007 as pre-crisis periods.

Table 7.2: Summary Statistics for Individual Sub-periods

Sub period	NumObs	Adj vol /transac.	Adj vol /day	Cond. Mean	Uncond. Mean	SD
1: (10/02/06-10/26/06)	19654	1.2012	1243	1.0332	4.0869	1.7609
2: (10/27/06-11/24/06)	22320	1.2983	1449	1.7122	2.1528	3.1556
3: (11/25/06-12/11/06)	10758	1.2258	942	1.6610	2.2410	2.8191
4: (12/12/06-01/03/07)	8861	1.1794	804	2.4234	3.0353	4.7939
5: (01/04/07-01/31/07)	20752	1.1016	1143	1.8147	2.0708	3.3973
6: (02/01/07-02/27/07)	16587	1.2090	1055	2.2048	2.4196	4.4210
7: (02/28/07-03/15/07)	32005	1.2426	3314	0.8157	0.9129	1.6208
8: (03/16/07-04/17/07)	25452	1.0911	1207	1.8094	2.0964	3.6274
9: (04/18/07-05/14/07)	23168	1.1125	1356	1.6049	1.9486	3.0250
10: (05/15/07-06/01/07)	19406	1.1503	1594	1.6575	1.6857	3.1579
11: (06/02/07-06/28/07)	33261	1.1208	1864	1.1403	1.3167	2.0851
12: (06/29/07-07/24/07)	25703	1.1980	1621	1.4093	1.7006	2.8473
Pre-crisis Average	21494	1.1776	1466	1.6072	2.1389	3.0593
13: (07/25/07-08/20/07)	66674	1.1952	3795	0.6188	0.7142	1.1143
14: (08/21/07-09/24/07)	54679	1.0751	2177	1.0156	1.0939	1.8984
15: (09/25/07-10/18/07)	30143	1.1184	1686	1.2766	1.3139	2.3225
16: (10/19/07-11/01/07)	18195	1.1335	2062	1.0539	1.3012	1.9716
17: (11/02/07-11/30/07)	57095	1.0279	2795	0.8163	1.0180	1.4498
18: (12/01/07-01/04/08)	48228	0.9838	2259	1.0508	1.1346	2.1391
19: (01/05/08-01/14/08)	11972	1.0376	2368	0.6162	0.9125	1.1118
20: (01/15/08-03/24/08)	167626	1.0151	3403	0.6718	0.9780	1.2264
21: (03/25/08-04/21/08)	40826	1.0120	2066	1.0348	1.1718	1.8516
22: (04/22/08-05/20/08)	32552	0.9402	1457	1.3312	1.3814	2.4188
23: (05/21/08-06/20/08)	40913	0.9232	1803	1.1986	1.3397	2.1436
24: (06/21/08-07/18/08)	52427	0.9168	2289	0.8501	1.0071	1.5314
25: (07/19/08-08/01/08)	19551	0.8633	1688	0.9878	1.1879	1.7272
26: (08/02/08-09/05/08)	43068	0.9094	1506	1.2155	1.3941	2.0992
27: (09/06/08-10/07/08)	92035	0.9915	3968	0.5397	0.7706	0.9348
28: (10/08/08-10/20/08)	56469	0.8819	5533	0.3385	0.4590	0.5341
29: (10/21/08-10/30/08)	35003	0.6952	2704	0.5182	0.5631	0.8582
30: (10/31/08-12/05/08)	83372	0.6601	2117	0.7055	0.8822	1.1667
31: (12/06/08-12/31/08)	28788	0.6032	914	1.3053	1.6255	2.7308
Post-crisis Average	51559	0.9465	2434	0.9023	1.0657	1.6437

Table 7.3: Individual Sub-periods WACD(1,1) Estimates (Oct. 06- Dec. 08)

	ω_0		γ_1		ω_1		α		v_1	
sub-period	estimate	t-stat	estimate	t-stat	estimate	t-stat	estimate	t-stat	estimate	t-stat
1: (06/10/02-06/10/26)	0.0280	9.9901	0.1098	28.0384	0.8831	238.1015	0.6853	157.8094	-0.0023	-4.6002
2: (06/10/27-06/11/24)	0.0623	15.6190	0.1091	26.2948	0.8618	182.7724	0.6891	171.4742	-0.0040	-14.4715
3: (06/11/25-06/12/11)	0.0731	10.4266	0.1175	18.1970	0.8498	110.5532	0.7153	118.3935	-0.0054	-8.0765
4: (06/12/12-07/01/03)	0.0609	9.2631	0.0999	16.8766	0.8788	135.7216	0.6512	101.8080	-0.0035	-10.8074
5: (07/01/04-07/01/31)	0.0944	16.0017	0.1146	23.1393	0.8378	130.3554	0.6739	156.1396	-0.0044	-16.5778
6: (07/02/01-07/02/27)	0.1590	14.4610	0.1426	20.5179	0.7898	87.7881	0.6134	136.9362	-0.0052	-13.0335
7: (07/02/28-07/03/15)	0.0153	17.0506	0.0858	35.0014	0.8970	320.0169	0.7682	211.8386	-0.0007	-11.8388
8: (07/03/16-07/04/17)	0.0956	18.1324	0.1279	24.4324	0.8244	137.9275	0.6664	170.7377	-0.0046	-28.2373
9: (07/04/18-07/05/14)	0.0842	16.5844	0.1105	25.3080	0.8427	150.7383	0.6806	168.2365	-0.0052	-10.9846
10: (07/05/15-07/06/01)	0.1089	15.6830	0.1407	23.0741	0.7938	98.4366	0.6615	147.4000	-0.0015	-14.5655
11: (07/06/02-07/06/28)	0.0526	20.0302	0.1010	31.9894	0.8572	203.0660	0.7291	203.0609	-0.0029	-239342
12: (07/06/29-07/07/24)	0.0261	14.3583	0.0791	27.4026	0.9049	277.9488	0.6954	179.2086	-0.0017	-21.8183
13: (07/07/25-07/08/20)	0.0120	25.4197	0.0733	47.9035	0.9104	514.1931	0.8138	311.8028	-0.0008	-24.4770
14: (07/08/21-07/09/24)	0.0275	23.7393	0.0929	42.4518	0.8826	344.2790	0.7426	267.7906	-0.0014	-34.1324
15: (07/09/25-07/10/18)	0.0600	18.5089	0.1088	29.2944	0.8405	170.6111	0.7004	187.4568	-0.0016	-29.8313
16: (07/10/19-07/11/01)	0.0372	14.8139	0.0992	24.6727	0.8723	184.7440	0.7415	150.3365	-0.0030	-17.8537
17: (07/11/02-07/11/30)	0.0179	22.0982	0.0893	44.0497	0.8927	402.3277	0.7832	296.7358	-0.0011	-25.0717
18: (07/12/01-08/01/04)	0.0483	25.4669	0.1195	42.2248	0.8382	238.8241	0.7525	263.3306	-0.0025	-36.6114
19: (08/01/05-08/01/14)	0.0114	29.0253	0.0897	69.5359	0.8961	635.0735	0.8529	404.2965	-0.0009	-19.7976
20: (08/01/15-08/03/24)	0.0074	29.4526	0.0874	90.6626	0.9061	939.4901	0.8352	517.8482	-0.0007	-18.9553

	ω_0		γ_1		ω_1		α		v_1	
sub-period	estimate	t-stat	estimate	t-stat	estimate	t-stat	estimate	t-stat	estimate	t-stat
21: (08/03/25-08/04/21)	0.0464	22.3040	0.1105	36.9615	0.8487	220.6562	0.7647	237.7052	-0.0021	-36.0872
22: (08/04/22-08/05/20)	0.0675	19.7127	0.1231	32.2059	0.8272	164.7782	0.7208	207.6561	-0.0018	-31.1245
23: (08/05/21-08/06/20)	0.0556	23.4985	0.1156	37.9921	0.8415	217.8162	0.7518	248.7474	-0.0022	-37.3989
24: (08/06/21-08/07/18)	0.0269	23.7601	0.0931	43.8971	0.8793	339.2705	0.8015	273.5940	-0.0018	-22.1509
25: (08/07/19-08/08/01)	0.0595	18.4887	0.1303	29.5244	0.8130	139.0447	0.7777	181.3284	-0.0022	-19.1306
26: (08/08/02-08/09/05)	0.0577	23.5995	0.1084	36.7144	0.8497	227.9471	0.7577	242.5063	-0.0035	-35.7727
27: (08/09/06-08/10/07)	0.0064	27.9297	0.0912	66.5433	0.9011	650.4943	0.8738	391.5802	-0.0005	-26.3402
28: (08/10/08-08/10/20)	0.0043	20.3496	0.0752	52.9102	0.9160	599.8348	0.9508	325.7005	-0.0004	-16.3445
29: (08/10/21-08/10/30)	0.0137	20.0871	0.0850	36.2771	0.8912	309.9321	0.8587	235.7639	-0.0008	-37.9838
30: (08/10/31-08/12/05)	0.0126	25.9003	0.0776	52.9023	0.9079	544.1041	0.8245	351.4213	-0.0014	-20.7188
31: (08/12/06-08/12/31)	0.01416	13.3841	0.1001	34.6757	0.8927	317.7301	0.7635	200.5283	-0.0013	-8.5350

Mean Shifts

In last subsection, we mentioned that the average conditional and unconditional mean of duration decreased after 24th July, 2007. In particular, the conditional mean has decreased from 1.6 to 0.9 seconds, and the unconditional mean also halved from 2.1 to 1.1 seconds. This downwards shift in the mean duration is well reflected in the declines of the constant term ω_0 in ACD model, since this fixed parameter of WACD model directly affects the mean level of expected durations. In Figure 7.7, large fluctuations of the parameters are shown before the most significant break A1, which is the bold grid line located between periods 12 and 13 (period 13 is the period immediately following 24/07/2007). The standard errors are also much bigger as represented by the longer band intervals shown in Figure 7.7. After the GFC was triggered in mid-July, 2007, consistent with the break at period 13, the level of ω_0 is smaller than previously with tighter standard errors. Although tighter standard errors could be associated with the increase in the level of trading activity, it also provides an indication of a more homogenous sample distribution. The decline in standard deviation and increased number of observations after period 13 are both responsible for the decline in the standard errors. At this stage, it is unclear whether there is a change in the form of distribution of the data by looking at ω_0 alone.

Interestingly, the smallest values of ω_0 are recorded in periods 7, 13, 20, and 28. These periods are the exact periods follow immediately after some of the most significant breaks, namely B1, A1, C3, and B2 in Figure 4 shown in last subsection. This is another clear indication of sharp drops in the mean level of expected durations right after major events, leading to changes in structure of the ACD model. It is reasonable to presume that investors quickly make their adjustments according to the information available. These actions cause increased level of trading activity and possible price adjustments. The level of price and trading intensity calms down when everyone is informed and asset prices have fully adjusted. This is consistent with the market microstructure literature, such as Easley and O'Hara (1992) and Easley et al. (1996).

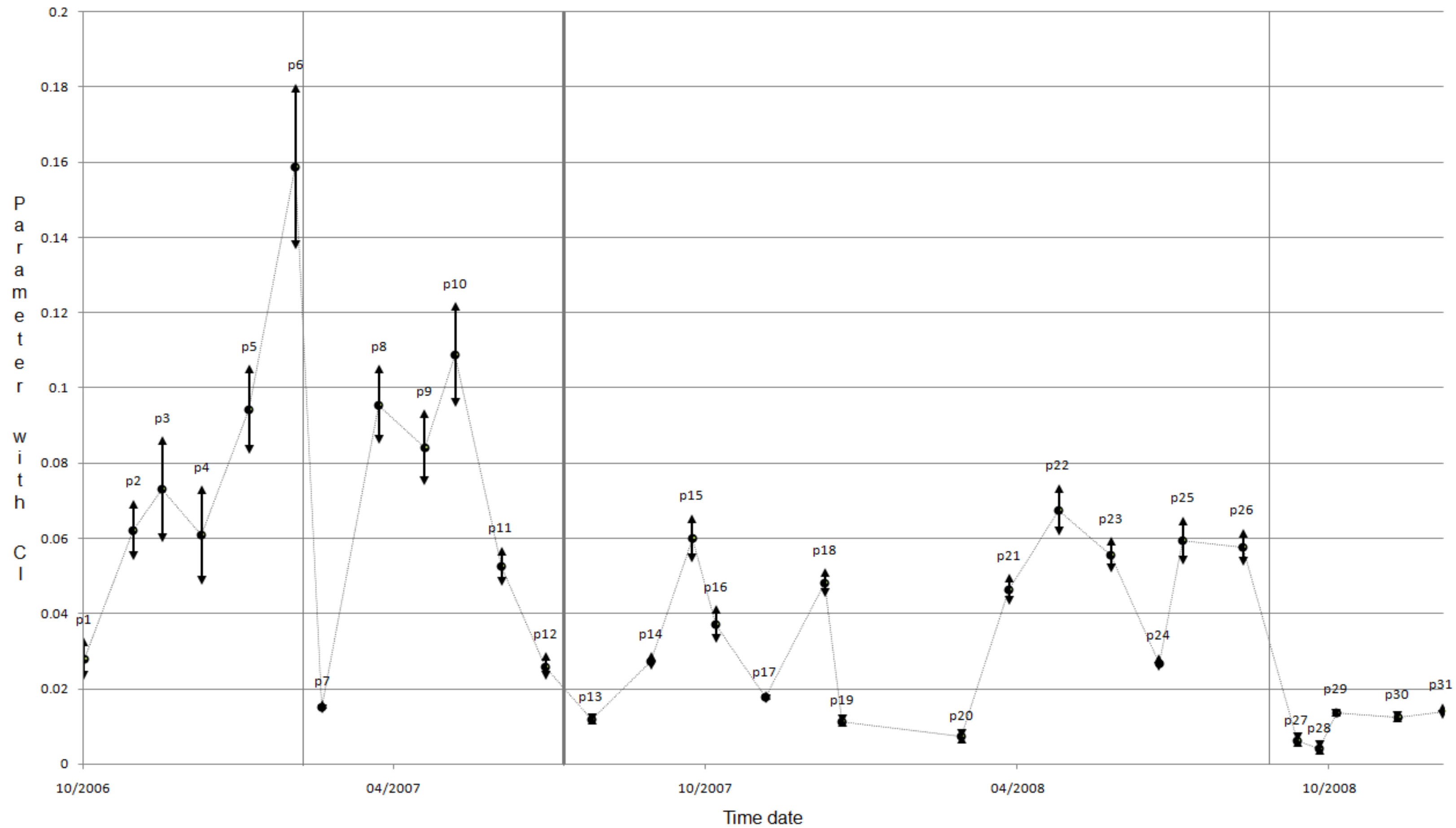

Figure 7.7: Estimated Parameters ω_0 with ± 2 Standard Error Confidence Interval for 31 Sub-periods of S&P 500 Duration Data. (Three grid lines indicate the three most significant break points, the bold grid is the most significant break point at A1, this also applies to the rest of parameter graphs)

Changes of Expected and Observed Durations

Changes in the past observed durations parameter γ_1 and past expected durations parameter ω_1 are shown in Figures 7.8 and 7.9. The loadings are slightly higher for parameter γ_1, and lower for ω_1 in the pre-crisis periods. The S&P 500 trade durations are more dependent on the past observed durations, and less dependent on past expected durations before the crisis. After the crisis was triggered, duration depends less on the past observed durations, but more on the past expected durations. This is explained by the more predictive and more clustered market after the crisis, since investors all consistently started to lose confidence in the market after the crisis. This is especially the case after September 2008 (around period 28), when the ω_1 parameter is very close to 1. In this case, the next duration is almost solely dependent on the past expected duration regardless of the past observed duration. In this case it suggests that in the time when an economic event occurs, durations experience less clustering as the dependence from past observed durations is lower. This is consistent with the analysis from diurnal patterns in Figure 7.3. The graphs of γ_1, and ω_1 in Figures 7.8 and 7.9 are almost symmetric, indicating a negative relationship exists between the past observed and past expected duration parameters.

Movements of γ_1 along the sub-periods are generally positively related to the ω_0 movements in last subsection. Lower dependence on observed durations corresponds to lower magnitude of the fixed components ω_0 in the ACD model from equation (7.1). The past expected duration parameter ω_1 also reaches its maxima in periods 7, 13, 20, and 28, which correspond with the the exact minimum ω_0 sub-periods in Figure 7.7.

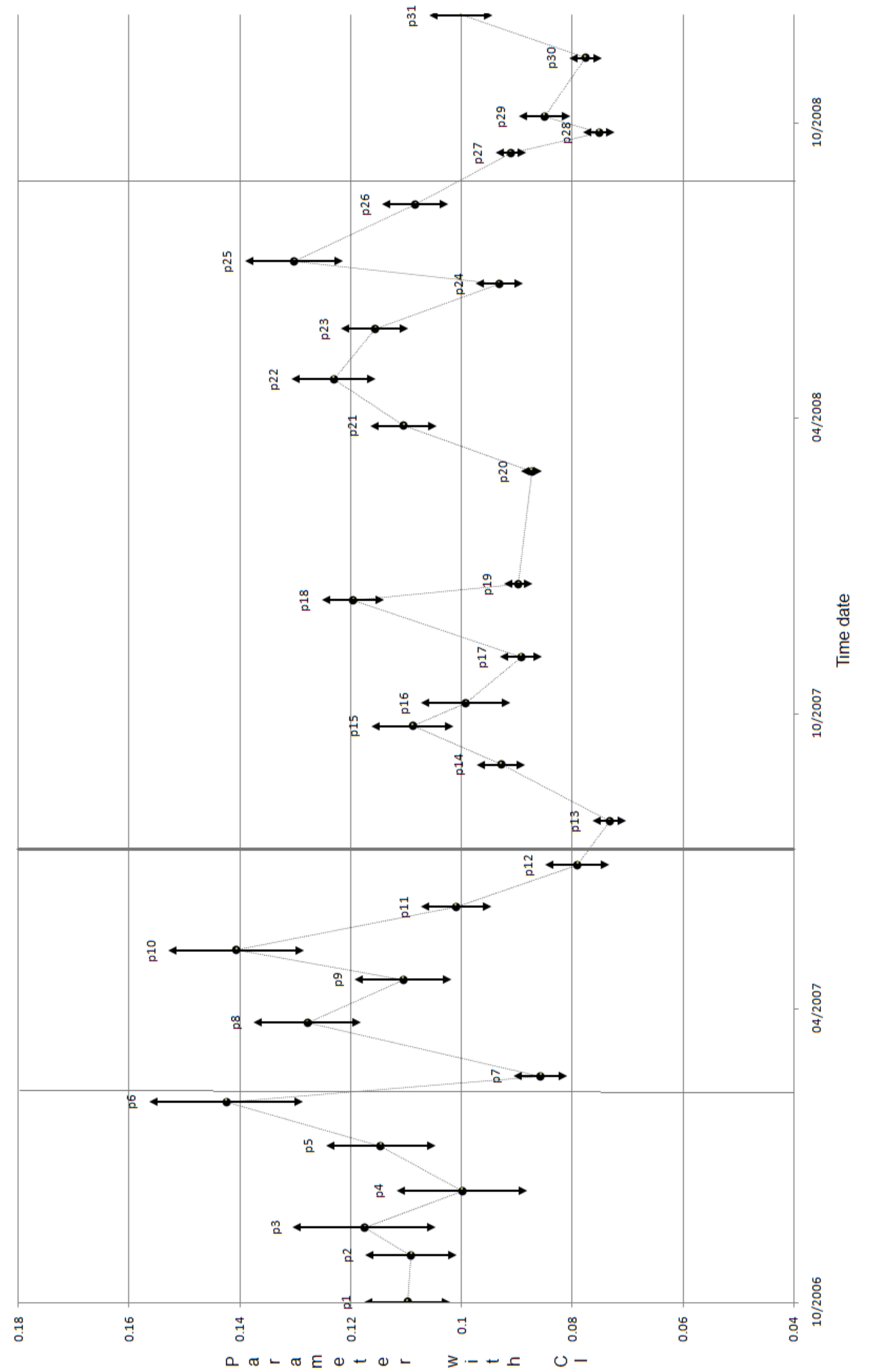

Figure 7.8: Estimated Parameters γ_1 with ± 2 Standard Error Confidence Interval for 31 Sub-periods of S&P 500 Duration Data.

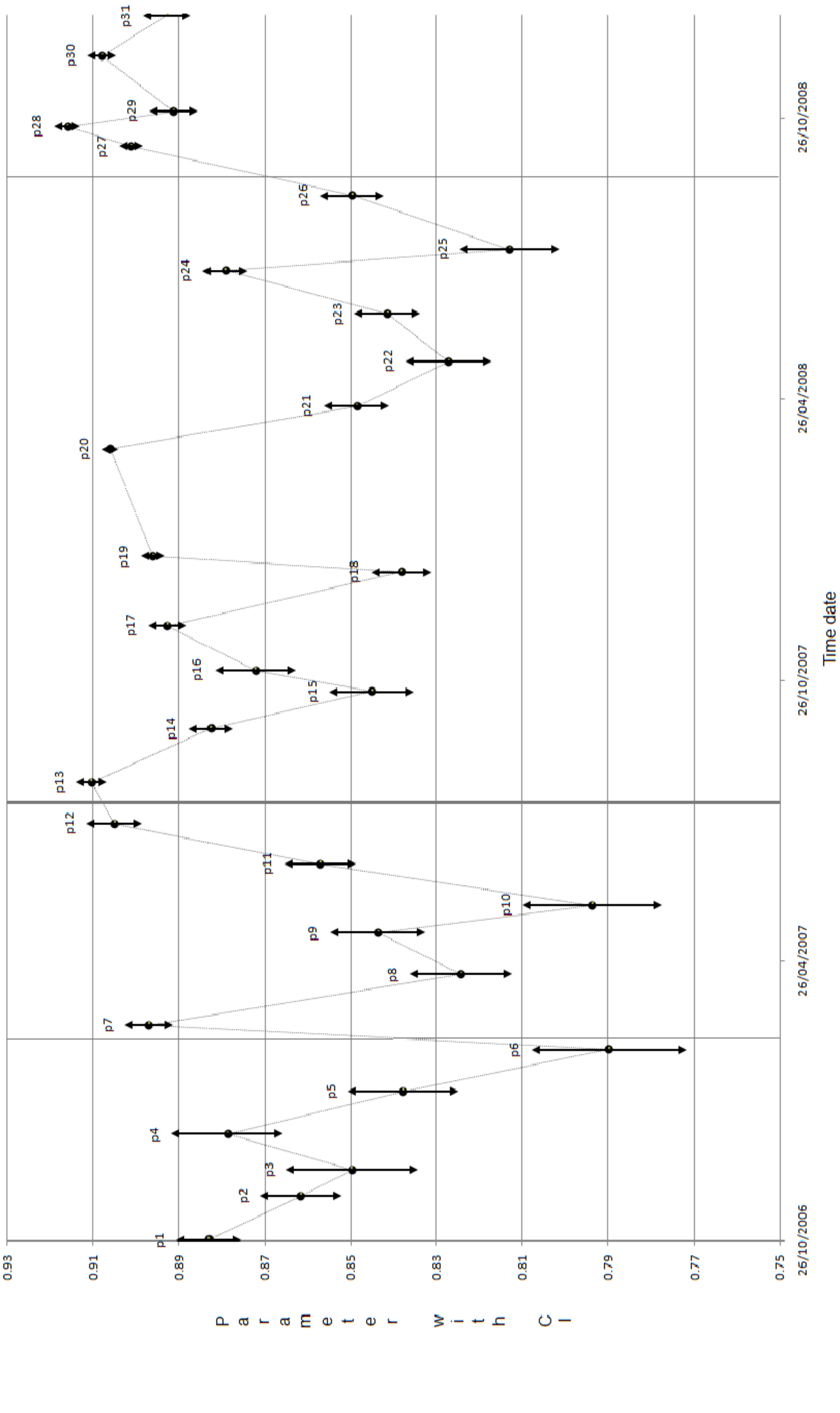

Figure 7.9: Estimated Parameters ω_1 with ± 2 Standard Error Confidence Interval for 31 Sub-periods of S&P 500 Duration Data.

Changes in Distributional Form

The shape parameter, α, in all sub-periods in the pre-crisis periods is in the range between 0.67 to 0.72. From Figure 7.10, immediately following period 13, α jumped to 0.81, indicating that the shape of the distribution is changing dramatically. If we examine the same period, period 13 in Table 7.2, the conditional mean duration is 0.61 and unconditional mean is 0.7361, both well below the overall average of 1. According to Easley and O'Hara, short durations imply a high level of trading activities and asymmetric information.

To get a better idea of implications of different α values, Figure 7.12 shows the probability density functions for $\alpha = 0.4, 0.6, 0.8$, and 1. In the estimation results, the shape parameter reached 0.95 in period 28, implying that the distribution in this period is close to a standard exponential form. The huge increase in α means that the waiting time of transactions becomes more exponential and durations are more homogeneous when events occur. On the other hand, huge increase in volume trade during crisis period may also contribute to the change of data probability distribution function. This is discussed in detail in the following subsection.

Recall that in the relatively calm period of the data set from 2004-2006 in chapter 6, the shape distribution parameters changed very little between the range of 0.63 to 0.73 from the same Weibull ACD (1, 1) estimates with structural breaks. The crisis period data set implies the presence of more dramatic changes in the shape of distributions for sub-periods. Ignoring possible changes in the distribution forms, especially during a volatile period may lead to biased estimation of ACD model parameters.

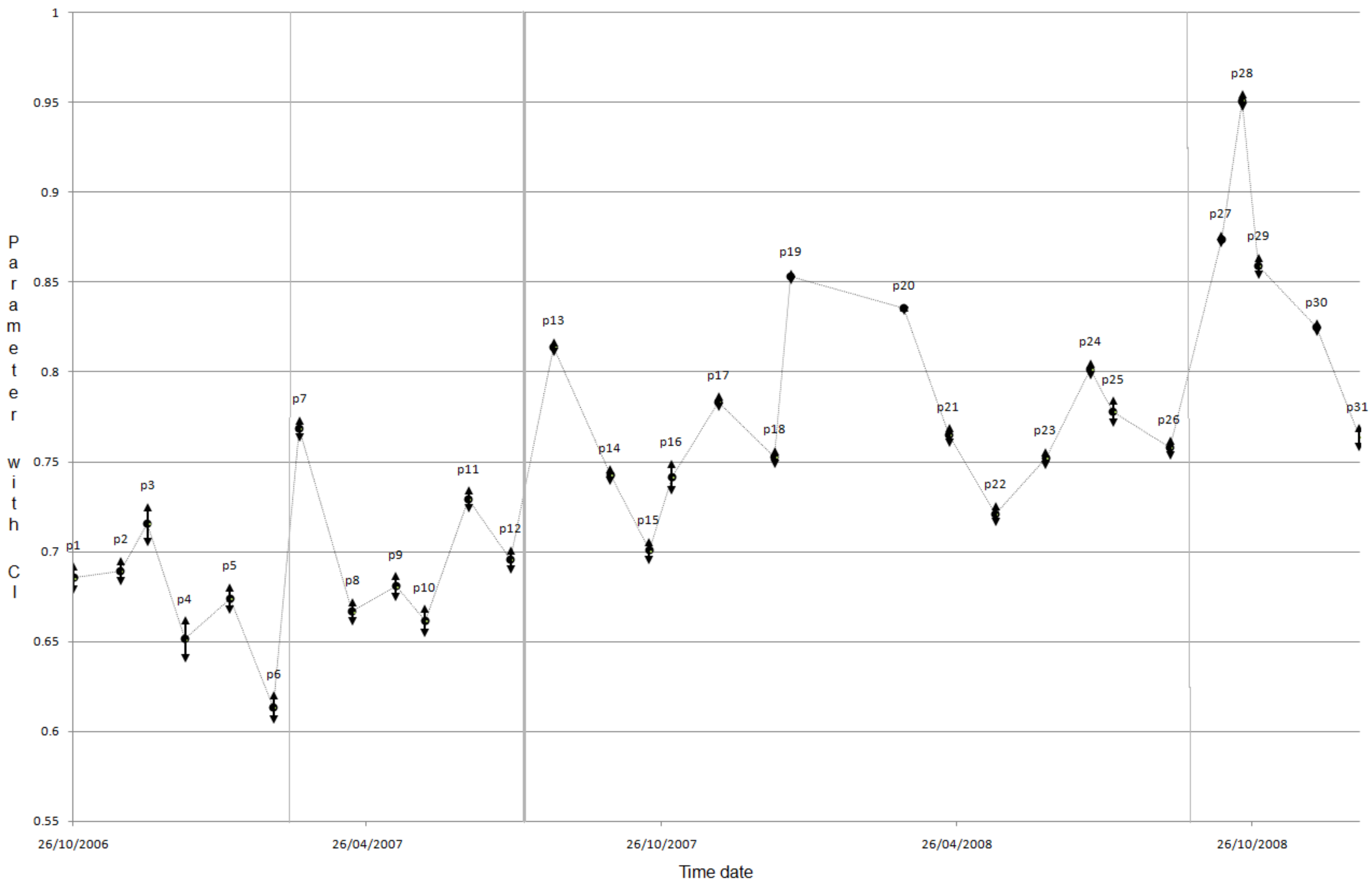

Figure 7.10: Estimated Parameters α with ± 2 Standard Error Confidence Interval for 31 Sub-periods of S&P 500 Duration Data.

Volume Effects

The influence of volume also changes dramatically over the sample period. Earlier, Figure 7.1 showed the huge increase in volume during this sample period. Figure 7.11 further shows the details of the parameter v_1, which changes over the sub-samples. The fact that volume coefficients are negative is consistent with the traditional market microstructure theory, where higher volume leads to smaller duration. From Figure 7.11 it is shown that the absolute value of parameter v_1 is smaller after the crisis was triggered. Some of the maxima of $|v_1|$ occurred in periods 7, 13, 20, and 28, suggesting that negative volume effects are smaller in the most heavily traded periods after structural breaks. Note that the traditional microstructure theory assumes shorter duration indicates higher volume. The volume parameter results in this chapter further suggest that volume effects become less important in the extremely heavy trading sub-periods.

One possible explanation for this phenomenon is that at the most volatile time, with stress and uncertainty permeating the market, investors may just trade purely for liquidity regardless of the volume. This is consistent with average volume statistics from Table 7.2. The average adjusted volume per trading day has increased from 1466 per day in the pre-crisis period to 2434 per day in the post-crisis period. The average adjusted volume of transactions per day increased dramatically immediately after a number of the major breaks, especially evident for sub periods 7, 13, and 28. However, at the same time the average volume per transaction decreased from 1.1776 in the pre-crisis period to 0.9465 in the crisis period. Trades are more clustered and there is greater dependence on observed duration in the pre-crisis period, where traders wait for information to arrive. This gives time for volume to build up and hence higher volume per transaction. In the period of market stress, duration tends to follow an exponential distribution, as discussed in last subsection, trades are less clustered. More trades are transacted for the purpose of liquidity, hence we observe lower volume per transaction but higher volume per trading day. In general, all parameters from pre-crisis sub-periods experience larger standard deviations than in the crisis periods, this might due to small number of observations for each subperiods in the pre-crisis period.

The analysis of the changes in the parameters in this section suggests that there are significant alterations to the duration process in crisis periods. Using sub-samples identified by an endogenous break point test we find that in periods of particular stress the dependence of expected duration on past observed duration decreases while the dependence on past expected duration rises. At the same time, the effect of volume on duration declines in impact. Over the period of the crisis, the distribution of the unconditional duration tends closer to an exponential, which is consistent with increasing homogeneity of trade. The results strongly suggest that applying duration analysis to sub-periods may give important insights into the prevailing trading dynamics which will be undetected if structural breaks are ignored.

Interestingly, our parameter analysis also identifies a pre-crisis fluctuation phase (periods 8 to 12), and a worst phase (period 28) during the 2007-08 financial crisis. The pre-crisis fluctuation periods start with the Shanghai Stock Exchange wobble in early 2007, while the worst phase corresponds to the rescue package from US Congress and interest rate cut by 6 of world leading central banks in early October 2008, right after break point C4. Compared to other periods within the sample, period 28 produces smallest ω_0, v_1, and largest ω_1, and α. These suggest that during this period, average waiting time between trades is extremely short and became less dependent on volume; meanwhile past expected duration plays the dominant role in determining expected durations, and the α parameter rises to 0.95 which is very close to an exponential distribution.

Some of the significant break points, such as A1, B1, B2 and C4 produce an insightful story of the build-up, and different stages of the crisis from the perspective of duration. The duration process is clearly informative regarding to the intensity of information arrival in the market, which is consistent with the proposal in the beginning of this paper. This paper provides an analysis of the 2007-08 crisis from a new angle, in a complementary approach to the traditional price adjustment research agenda. Through our analysis of the WACD model parameters, the occurrence and intensity of trade is indicative of different levels of information arrival in the market. A complex ACD model specification with static parameters may be developed to cover the whole sample period in this paper, however, the study of individual subsamples clearly unveils further valuable information from duration process. Such information is normally ignored by neglecting structural breaks within the sample period.

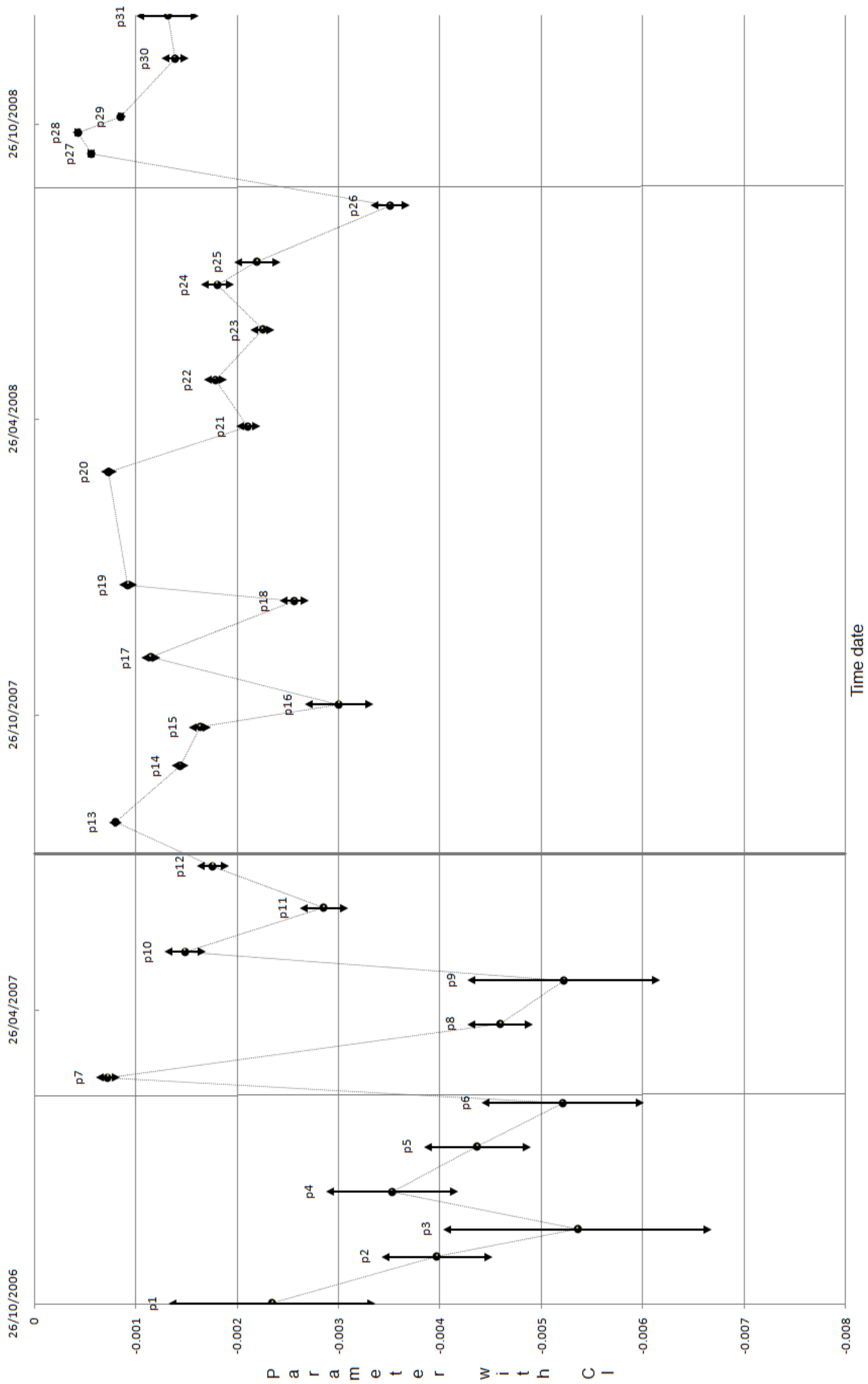

Figure 7.11: Estimated Parameters v with ± 2 Standard Error Confidence Interval for 31 Sub-periods of S&P 500 Duration Data.

7.4.3 Relationship with the Market Events

Many of the structural changes in ACD model parameters can be aligned with events happened during the crisis period. Some of the economic events between 2007 and 2008 are listed in Table 7.4. In this subsection we align some of the most significant breaks with economic events occurring in the same periods.

The most significant break point, A1, occurred on 24th, July 2007, right after the collapse of two of Bear Stearns hedge funds. Mid-July 2007 is generally considered as the starting date for GFC by many economic analysts, such as Almunia et al. (2009) and Brunnermeier (2009). In the month of July, 2007, the Dow Jones Industrial Average (DJIA) closed above 14,000 points at record high in the history at that time. The high level of DJIA points reflects bullish market with an increasing price overall. Some further economic events between 2007 and 2008 are listed in Table 3. Break point B1, which occurred on 27 February 2007, captures the date when Chinese stock market wobble took place. At that time the Shanghai Stock Exchange experienced its largest drop in 10 years, triggered major drops in worldwide major stock markets. Some other significant breaks worth looking at, in particular are breaks B2 and C4 occur during the turbulent period in September and October 2008. Breakpoint B2, the biggest structural change in the crisis period, marks a change after 5 September 2008. This is consistent with the announcement of the takeover of Fannie Mae and Freddie Mac on 6 September, which is followed by the Lehman Bros bankruptcy and AIG rescue in the immediate period. This period, until the 7 October 2008 is not particularly long but incorporates major disruptions and most of the key US crisis events. The end of this period, which is C4, is marked by the marked easing in monetary policy around the world. The next period, labelled period 28, runs from 8 October 2008 to 20 October 2008, a very short time span, but which importantly incorporates the annoucements of bank recapitalisation plans around the world by major governments, including the US, UK, Switzerland, France, Germany and the Netherlands and general statements that systemically important banks would not be allowed to fail; see King (2011).

The effects of these events on duration modelling is significant. Each of the loadings on observed and expected duration, as well as the mean level of duration and volume impacts differ following these events. Clearly the structure of conditional duration is affected by these economic developments, providing important evidence that extreme market conditions, and policy reactions to them, directly effect revealed market microstructure.

Table 7.4: Major World Events Time Line During 07-08 Crisis Period

2007	February 27	Chinese stock wobble, the Shanghai Stock Exchange Composite Index tumbles in its largest drop in 10 years.
	April 2	New Century Financial, one of the U.S. largest subprime lenders files for bankruptcy court protection.
	June	Two hedge funds of Bear Stearns run into large losses and force major Wall Street firms to seize loan involved assets.
	July 17	Two Bear Stearns hedge funds collapsed.
	August 9	French investment bank BNP Paribas suspends three investment funds invested in subprime mortgage debt, the European Central Bank pumps 95 billion Euros of intervention into the European banking market.
	August 17	The U.S. Federal Reserve cuts discount lending rate by half a percent.
	September 14	British bank Northern Rock suffers a bank run after approaching the Bank of England for a loan facility to replace money market funding.
	September 18	The U.S. Federal Reserve starts cutting interest rates and agrees to start loaning money directly to Wall Street firms.
	Mid October	A consortium of U.S. government backed banks announces a super fund of $100 billion to purchase mortgage-backed securities in the subprime crisis.
	December 24	The consortium of banks officially abandons the mortgage crisis bail-out plan announced in mid October.
2008	January	Stock market downturn
	March 16	JPMorgan Chase & Co. acquires troubled Wall Street firm Bear Stearns at $2 a share backed by the Federal Reserve.
	September 6	Treasury Secretary Henry Paulson announces a takeover of Fannie Mae and Freddie Mac.
	September 15	Merrill Lynch is sold to bank of America and Lehman Brothers collapses.
	September 17	The US Federal Reserve lends $85 billion to American International Group to avoid bankruptcy.
	October 3	President George W. Bush signs the Emergency Economic Stabilization Act, with a $700 billion Troubled Asset Relief Program to purchase failing bank assets.
	October 8	Central banks in USA, England, Canada, Sweden, Switzerland and the European Central Bank reduce interest rates by half a percent.
	October 14	US announces that TARP funds will be available for bank recapitalization.
	November 15	G-20 Washington Summit on Financial Markets and the World Economy which achieved general agreement amongst the G-20 on cooperating in key areas to strengthen economic growth, and deal with the financial crisis.
	November 24	The U.S. government agrees to rescue Citigroup providing a package of guarantees, liquidity access and capital.

*Events are compiled by the author from online news sources such as Financial Times, Reuters, and New York Times.

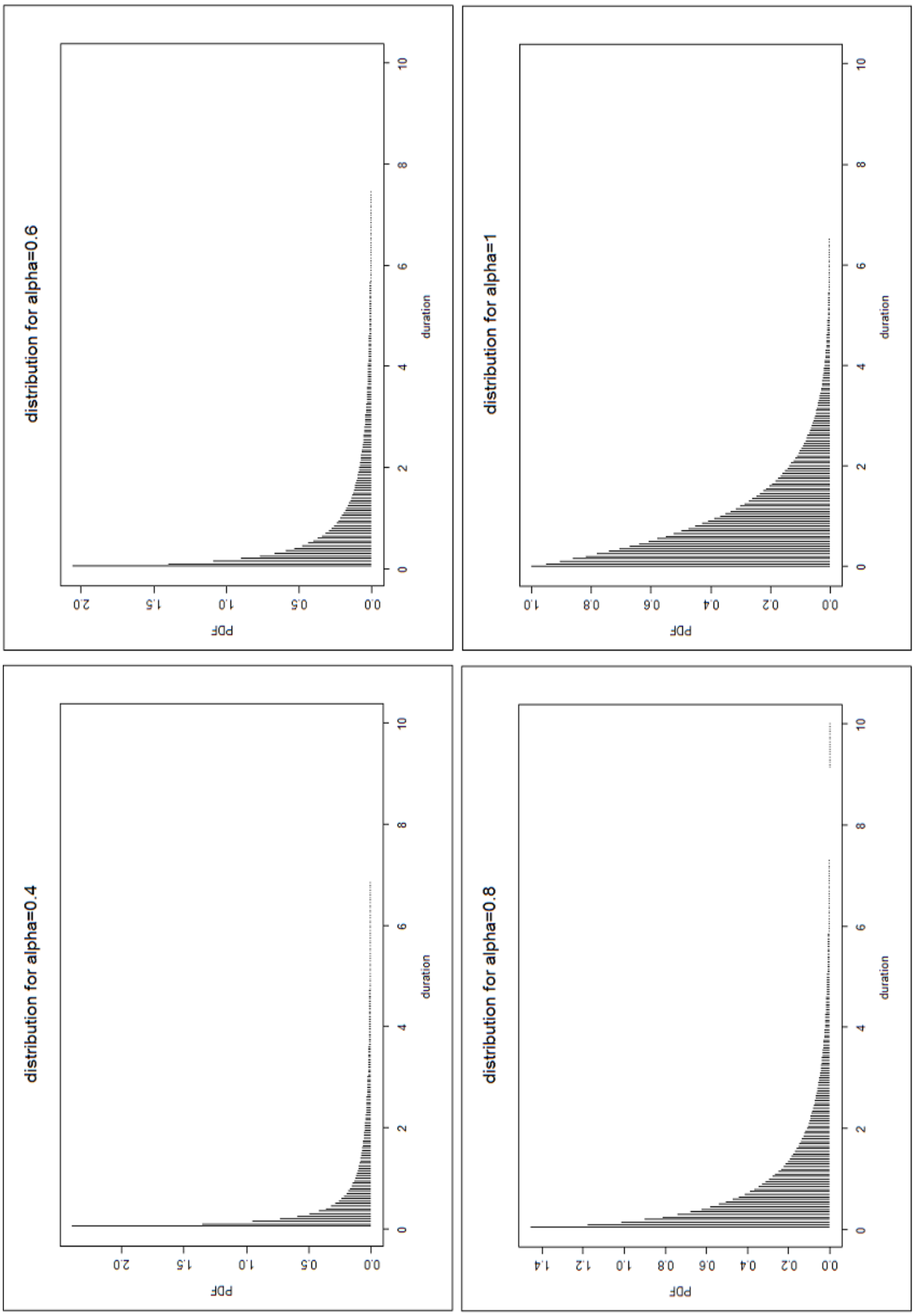

Figure 7.12: Distribution Shapes with α Range from 04 to 1.0

7.5 Conclusions

In this chapter, the Andrews and Ploberger Structural break tests are performed based on a WACD (1,1) model over the global financial crisis period of Oct. 2006 to Dec. 2008. 30 breaks were detected using a 22 day filter. During mid-2007, there is a major market environment change due to the collapses of major Wall street financial institutions, leading to a new structure. This major structural change is exactly captured as the most significant change in our WACD model. Some of the most significant change points are consistently aligned with the economic events in the same periods. Incorporating these events with our ACD estimations in sub-periods, yields a better understanding of the S&P 500 trade duration process in the electronic futures market.

Compared with the non-crisis period of the market, the trade durations for S&P 500 are found to be less clustered, especially in the crucial times of the crisis. During the crisis investors worldwide traded heavily through the after-hours electronic market, which makes the market duration and volume diurnal pattern start to change.

The WACD model parameters of all sub-periods reflect the development of the crisis. All findings are consistent with market microstructure except volume parameters are found to be positively related to duration in the most critical times of the crisis.

Chapter 8

Conclusions

8.1 Introduction

This thesis examined the after-hours electronic futures market before and during the recent (2007-2008) global financial crisis period from the perspective of trade durations. Under market microstructure theory, duration and volume play important roles in determining price adjustment and market efficiency. Papers such as Easley and O'Hara (1992), O'Hara (1995), and Madhaven (2000) also stress the information content of duration.

There have been rapid developments in after-hours electronic markets in the last 10 years. With the increased availability of intraday high-frequency data sets duration modelling has become feasible for these new products. The after-hours electronic markets have received little attention in duration modelling in the financial time series literature, justifying our empirical contribution. In this thesis we applied a class of models for trade durations in the after-hours electronic market for U.S. equity futures. Using data for the pre-crisis period (2004-2006) and then data during the GFC period (2007-2008) allows examination of changes in the market behaviour with respect to trade duration.

The thesis began with a review of literature on ACD models. A review of structural break test studies is also included as structural changes are highly likely to occur in our data sample examined, especially for the data during GFC period. A background to the after-hours electronic equity futures markets and a brief overview of the causes and developments of the 2007-2008 GFC followed. The remainder of the thesis comprises of four papers. These papers cover applications of linear ACD models, threshold and logarithmic nonlinear ACD models, and structural breaks studies within the ACD framework based on the after-hours electronic futures market data. The lag structure of ACD models and the role of volume (as a mark) across different periods were also examined. The main findings from this thesis are summarized in the following sections.

8.2 Thesis Contributions

In this thesis we applied different forms of ACD models to the after-hours electronic equity futures market. The models examined include linear and nonlinear (logarithmic and threshold) forms of ACD models pursued to higher lag orders using Exponential, Weibull, or Generalized Gamma distributional assumptions. The thesis offered a detailed comparison study of the overall fit of the linear ACD models by allowing different forms of error distribution. The results from nonlinear logarithmic and threshold ACD models were compared to linear ACD models.

Apart from demonstrating the effects of error distribution assumptions on model fitting, the thesis also challenged the dominance of linear and logarithmic

ACD models in the literature based on an approximately two-year-long after-hours intraday data set. As this data set is much longer than data usually examined in the duration modelling literature, long memory, structural change, and nonlinearity problems arise. Although these problems are addressed in the existing ACD literature our much longer data set brings these problems to the fore.

In order to further address the nonlinearity within our long data set, we studied structural breaks within the estimated ACD models. The study empirically contributes to the financial time series literature by testing structural break effects in ACD models with application to the after-hours electronic equity futures markets. We present a comparison study between the Weibull ACD model with and without potential structural breaks.

The thesis also extends duration studies over the 2007-2008 global financial crisis using ACD models with potential structural changes. The Weibull ACD model applied to the GFC data sample captures the clustering of the duration in the after-hours equity futures market, and provides a sequence of structural changes which we posit arise from the market participants' trading behaviours. This thesis also contributes to the global financial crisis literature by analyzing structural changes during the crisis solely from modelling the time intervals between trades.

The detected structural breaks align with financial crisis timelines, picking up the most significant change point as 24th July, 2007, closely related to the dates associated with the start of the crisis in a now relatively large body of literature. The changes in parameters of the ACD model imply a decreased trade duration immediately after these large structural changes. Analysis from past observed and expected duration parameters suggests a lower degree of clustering after the major breaks. The duration error distributional shape also changes over the crisis. It is found that immediately following major structural changes, the waiting time between transaction behaves more exponentially and is more homogenous. By evaluating the information obtained from modelling duration, particularly under potential structural breaks in the market, this thesis confirms the market microstructure theory, as suggested by Easley and O'Hara (1992), that duration between trades contains valuable information.

8.3 Main Findings

The main findings of the four papers embedded in this thesis are presented in chapters 4, 5, 6, and 7. In chapter 4, linear ACD models with Exponential, Weibull, and Generalized Gamma distributional assumptions on error terms are examined based on NASDAQ and S&P 500 data sets. Trade durations from both data sets experience long memory, low autocorrelation, and strong clustering, but the S&P 500 data requires a more complex model due to large duration observations. For the NASDAQ duration data, a Weibull ACD (5, 5) model produced the best estimate results in terms of removing residual serial correlations. The S&P 500 data set contains much larger observations and experiences convergence problems when allowing more general forms of error term distribution assumptions. Also in chapter 4, we presented a threshold approach to the S&P 500 data and implemented a threshold Weibull ACD (4, 1) model. The addition of volume to the ACD model in both data sets captures statistically significant negative relationships between conditional expected duration and volume. This negative relationship is consistent with the market microstructure theory.

Although the threshold Weibull ACD model produces the best estimates for the S&P 500 data set in chapter 4, it also indicates that nonlinearity and structural effects should be considered more carefully. The S&P 500 data also failed to pass the F-test of linearity, further indicating the necessity of a nonlinear approach. For this reason, in chapter 5 we applied nonlinear logarithmic ACD models on the same S&P 500 data set. A logarithmic Weibull ACD (2, 2) model with volume as an additional mark is found to be the best logarithmic ACD model. The logarithmic ACD models are shown to be capable of handling a certain degree of nonlinearity. Moreover, compared with threshold ACD models, the logarithmic ACD models are far less costly to estimate. In chapter 5, it is shown that by taking nonlinearity into account, the model estimates can be greatly improved. However, the over-dispersion ratios from the logarithmic ACD model for S&P 500 data imply that the residuals are still over-dispersed.

Apart from nonlinearity, another characteristic of the intraday high-frequency data is the long memory in the autocorrelations, which could be caused by ignoring structural breaks within the model. Therefore in chapter 6, we focused on the potential for structural breaks within ACD models using the same S&P 500 data as in chapter 5. The Andrews and Ploberger structural break tests applied to the Weibull ACD model found 19 break points during the period July, 2004 to September, 2006. This period is consequently divided into 20 sub-periods which were modelled individually using a Weibull ACD (1, 1) model. Detailed model parameters and summary statistics are summarized for each individual sub-period. The results in chapter 6 showed that by allowing structural breaks, the overall model fitting can be further improved. Market movements in duration are better captured from studying duration models in the individual sub-periods. The individual sub-periods parameters are compared and plotted in this chapter.

Finally the S&P 500 data set during the 2007-2008 global financial crisis period is investigated in chapter 7. The recent global financial crisis has been analyzed by many economists. However, the work in this thesis provides an unique analysis of the crisis through the lens of trade durations. In chapter 7, structural breaks using a Weibull ACD (1, 1) model are tested by applying break tests from chapter 6. The data set examined in chapter 7 is further complicated not only because the financial crisis took place in mid 2007, but also contains the merger of the COBT electronic market with the CME in early 2007. There were 30 break points identified and the most significant break was found to be on 24th July, 2007, a date which aligns perfectly with anecdotal assessments of the onset of the global financial crisis. The 31 individual sub-periods were fitted with Weibull ACD (1, 1) models and the parameter statistics were summarized. Each of the Weibull ACD parameters, namely the constant, past raw durations, past conditional expected durations, volume, and distribution shape parameters were plotted and studied across its sub-periods. The changing distribution shape parameters across different sub-periods, between events indicates possible shifts in the actual error distributions. The changes in distributional form will not be picked up if one ignores the structural changes. In chapter 7, events during the crisis period are studied and related to the parameter changes across different sub-periods. The results in the parameters analysis showed that the crisis events can also be studied from trade durations. Results from this chapter further demonstrate that the ability of trade durations to reflect market information.

Overall, we found that the traditional linear ACD models from Engle and

Russell (1998) struggle to handle relatively long period duration data in the after-hours electronic futures market. The data nonlinearity and long memory problems evident in the data made the model residual serial correlations very difficult to incorporate with linear ACD models. By applying nonlinear ACD models such as threshold and logarithmic models, the overall estimation results improved significantly. Volume plays an important role in duration modelling and has an negative relationship with duration, consistent with market microstructure theory.

Results from the model estimates consistently suggest that the data sample from the after-hours electronic futures market experiences long memory and a large degree of clustering. Duration data from this electronic market using the GLOBEX platform is particularly complicated due to the following effects. Firstly, the overnight effects; both informed and uninformed traders have the opportunity to make adjustments to their portfolios in reaction to events which occur overnight, making the response to such events in this market almost immediate. Secondly, the news announcement effects; the electronic trading benefits informed traders who are able to take advantage of their available private information before news announcements in the day trading period. The anonymous nature of the market may provide further benefits in this context. Thirdly, the internationalized trading platform makes price adjustments in this market sensitive to events and news on a global scale. Finally, since the introduction of E-mini and E-micro versions of futures contract in this electronic futures market, small retail and liquidity traders have increased trade in these smaller value contracts, shrinking the relative importance of the standard contracts. Hence strong nonlinearity and structural effects are highly possible in the after-hours futures market.

The structural break effects study in chapter 7 applied to the S&P 500 futures data during the 2007-2008 GFC period suggests a high level of structural changes. The after-hours futures market during the GFC is very volatile, not only because of native U.S. events, but also policies and events from financial institutions worldwide. The model error distribution shapes differ across various structural periods, indicating possible behavioural changes in trading. Results from Weibull ACD models in individual sub periods indicate that times between trades are less clustered and more homogenous immediately after a significant break (event). Effects from trade volume to conditional expected duration becomes smaller immediately after an significant break point. It is possible this result is obtained due to the large scale of volume transacted, but the detailed reasons remain unclear, and provides scope for future research. Through duration modelling, the influences on the electronic market from GFC associated news events becomes clearer. Successful identification of distinctive structures from modelling durations further supports the importance of modelling duration in modern financial markets.

8.4 Future Extensions

The usefulness of linear ACD models may be limited, especially in liquid markets. This has been demonstrated from the NASDAQ and S&P 500 results in chapter 4. With the development of increasingly flexible nonlinear ACD models, the after-hours high-frequency data can be better modelled. For instance, in chapter 4, we attempted a threshold ACD (4, 1) model which provided a better estimate over linear models. This attempt also raised a question of whether the clustering of durations could be linked to sources in different time zones, such as trading activities from Europe, U.S., and Asia. Duration clustering in this after-hours market may be influenced by factors such as informed and uninformed traders coexisting, participants being anonymous, and other trading regulation restrictions[1]. Perhaps a more in-depth threshold ACD model study could provide more insight into this after-hours electronic market.

Research on ACD models has extended in many more directions, especially the regime smooth transition ACD models of Meitz and Teräsvirta (2006), latent factor models of Ghysels, et al. (2004), and the mixture distribution ACD models of Hujer and Vuletic (2004). The above more complex forms of nonlinear ACD model with greater flexibility are of prime interest. These models may improve model estimation for the after-hours electronic equity futures market data, although the more complex ACD models are more time consuming and involve heavy computational burden. In addition, more flexible error distribution assumptions could also improve duration modelling in this after-hours market. As the results in chapter 7 show, the error distributions could be changing, especially in the light of economic events and their ensuring structural changes, and a more flexible distributional assumption could improve the model. In this thesis, we restricted our attention to the Exponential, Weibull, and Generalized Gamma distributions, alternative distributional assumptions, such as Burr distribution, Generalized F-distribution, or even mixed distributions may yield further improvements in results over the after-hours market data, although at the cost

[1]Different stages of trading cycle in after-hours limits buy/sell and modify contracts. See chapter 3.

of additional computational intensity.

In a similar manner to the existing ACD literature, we also find that trade durations are difficult to model properly in terms of removing residual serial correlations. In future studies, price durations and volume durations could also be used to examine the after-hours futures market. It would be interesting to examine a comparison ACD model study based on the after-hours electronic market using all these three forms of durations. In addition, other marks associated with the transactions such as bid-ask spread and price could also be added to the model, although data availability is a limitation in the data set used for this thesis.

The structural effects within the ACD models analyzed in this thesis are limited due to detection filter restrictions. A less restrictive detection criteria may provide a wider range of structures over the global financial crisis period. The increased number of structures detected using a less restrictive filter may also provide the opportunity to evaluate a more complex regime transition ACD model to better understand the GFC period. The Weibull ACD (1, 1) model was used to study structural effects in this thesis, and more sophisticated models with flexible distributional assumptions may contribute to analyzing the structural changes and the after-hours financial markets more precisely.

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